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Search for a unified theory*

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[Abstract: Today we have an extremely successful theory describing the physics of elementary particles. This theory-known as the standard model-has been able to explain the result of almost all experiments involving elementary particles, and have also given rise to many new predictions which have subsequently been experimentally verified.

Despite the enormous success of the standard model, we do not yet have a unified theory describing all natural phenomena. The main obstacle comes from the difficulty in incorporating gravity within the framework of the standard model. Although significant progress has been made in this direction with the help of string theory, we are still far from achieving our final goal.]

In this talk I shall be addressing the question : What are we and everything around as made of? This subject has a long history, and in particular, was studied by the Greek philosophers more than 2000 years ago. In particular, a few of them, now known as the **at mists.** asked : what should happen if we take a lump of some material, and keep dividing it? They concluded that if we continue this division, we would reach a stage where we cannot divide the material any further. What we get at this stage will be the smallest unit of matter.

Our present knowledge indicates that they were probably right, but they had no way of knowing this, since they could not actually carry out this experiment.

After this brief historical note, I shall turn to our modern understanding of the ultimate constituents of matter. This has been depicted in Fig. 1. As we see from this figure, the primary building blocks of matter are molecules, which themselves are made of smaller objects, – the atoms. Each atom is made of a nucleus at its centre, and a set of electrons revolving around it. Our present knowledge indicates that electrons cannot be subdivided further, and hence can be called truly elementary particles.

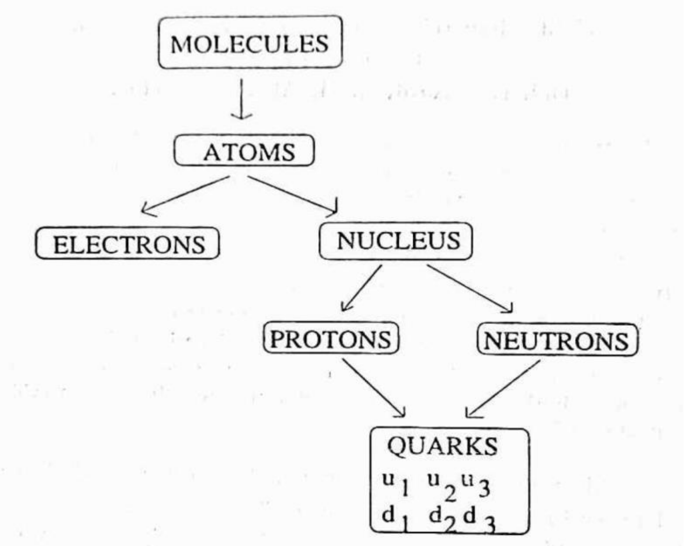

Fig. 1

Our modern understanding of the ultimate constituents of matter.

But nuclei are made of even smaller objects, – protons and neutrons. Each proton (and neutron) in turn is made of even smaller objects – the quarks. We find two kinds of quarks inside a proton and a neutron,– the up (*u-)* quark and the down $(d -)$ quark. Proton contains two up quarks and one down quark, while the neutron contains one up and two down quarks. Actually each quark comes in three varieties (colours). Thus there are three varieties of u quarks, which I shall label by u_1, u_2 and u_3 and also three varieties of d-quarks, labelled as d_1, d_2, d_3. All present day experiments have failed to show any substructure of quarks. Thus it appears that quarks, like the electrons, are truly elementary.

This however, is not the end of the story. In order to understand the various properties of matter, it is not enough to know the constituents of matter, we also need to know how these constituents interact with each other. This will tell us, for example, what keeps the constituents together inside an atom, molecule, nucleus, proton or neutron. This will also answer questions like : what happens when two particles (molecules, atoms, nucleus etc.) come close to each other. Such knowledge is relevant for understanding various reactions e.g., chemical reactions, nuclear reactions, etc.

What kind of interactions among matter do we observe in everyday life? There are two kinds :

1. **Gravitational interaction :** This is responsible for earth's gravity, planetary motion, etc.
2. **Electromagnetic interaction :** This is responsible for force due to a magnet, lightning, etc. However, what is not so obvious is that it is responsible for almost every interaction that we observe in everyday life which cannot be attributed to gravity. Thus it is electromagnetic interaction between us and

the earth which prevents us from falling to the centre of the earth under the action of gravity. electromagnetic interaction is responsible for light, radiowaves and other electromagnetic waves. Even sound waves travelling through a medium has its origin in the electromagnetic interaction between the molecules of the medium.

It turns out that in order to describe the interaction between the elementary particles, we need to include two other kinds of interactions :

1. **Strong interaction :** This is responsible for binding the quarks inside a proton or a neutron, and also for binding the protons and neutrons inside a nucleus.
2. **Weak interaction :** This is responsible for radio-active β-decays of nuclei as well as many other reactions involving elementary particles.

It turns out that in studying the physics of elementary particles, we can ignore the effect of gravitational force. To see this, one can compare the electrostatic force between two protons with the gravitational force between two protons. The result is

$$\frac{\text{Grav. Force}}{\text{Elec. Force}} = \frac{G m_p^2/r^2}{e_p^2/r^2} \sim 10^{-36}$$

where G is the Newton's constant, m_p is the proton mass and e_p is the proton charge. Thus we see that the gravitational force is really tiny compared to the electrostatic force. Similarly all other forces are also much larger than the gravitational force. Thus we shall ignore the presence of gravitational force for the time being, but shall return to it later in the talk.

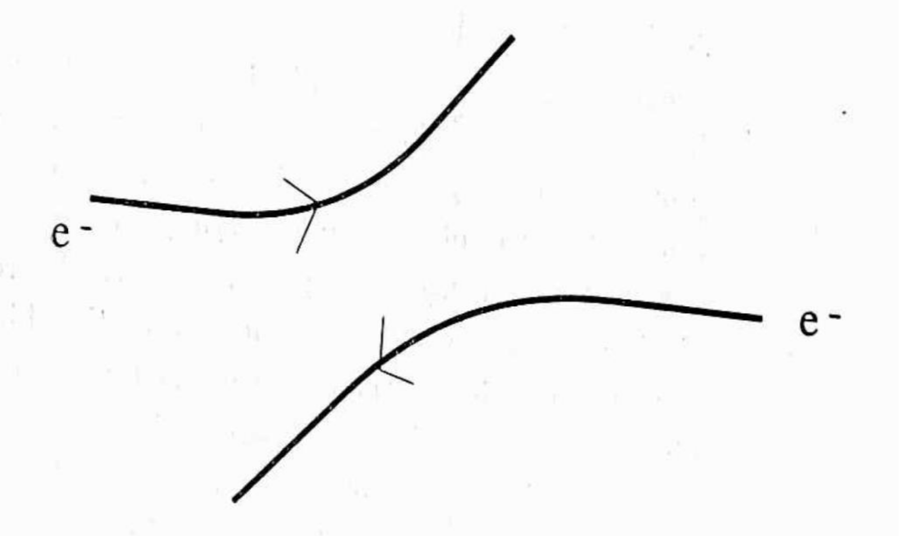

Fig. 2

Scattering of two electrons : Classical picture.

With the help of quantum theory one can give a unified description of elementary particles, and the interaction among the elementary particles. Consider, for example, the electromagnetic interaction between two electrons when they pass each other. Due to the interaction, each particle gets deflected from its original trajectory. In quantum theory, one provides a different explanation of the same phenomenon. According to this theory, the deflection takes place because the two electrons exchange a new particle, called photon, while passing near each other. The photon is capable of carrying some amount of energy and momentum from the first electron to the second electron, thereby causing this deflection. We call the photon the mediator of electromagnetic interaction.

In this language, we can describe an interaction by specifying the particle(s) which mediate the interaction. Thus for example :

- Strong interaction is mediated by eight different particles known as gluons. These particles are all electrically neutral.
- Weak interaction is mediated by three particles, denoted by $W^{\pm}$ and Z. $W^{\pm}$ carry $+1$ and -1 unit of electric charge respectively, whereas Z is electrically neutral.

Thus we must add these particles, as well as the photon, to our list of elementary particles. But this still does not exhaust the list of all elementary particles. One often finds new elementary particles in cosmic rays, during the decay of unstable particles (e.g., the neutron), or when one makes two elementary/composite particles collide with each other. Also, mathematical reasoning shows that for every particle there is an anti-particle which carries charge opposite to that of the particle. All of these new elementary particles must

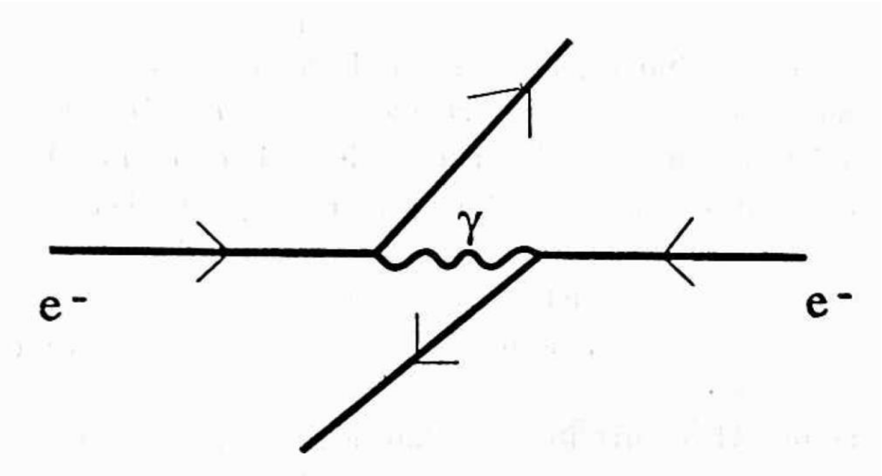

Fig. 3

Scattering of two electrons : the quantum picture.

also be added to the list. We shall now give a list of known elementary particles in nature:

List of known elementary particles

Quarks :

$u_1, u_2, u_3 \quad d_1, d_2, d_3 \quad c_1, c_2, c_3$

$s_{1,}, s_2, s_3 \quad t_1, t_2, t_3 \quad b_1, b_2, b_3$

Leptons

(e, v_e), (μ, v_μ), (τ, v_τ)

Mediators

Gluons: $g_1, \dots, g_s$ Proton: γ

$W^\pm$, Z

Note that we have divided the particles into three classes – quarks, leptons and mediators. We see from the list that besides the up and the down quarks, we have four more kinds of quarks, charm (c), strange (s), top (t) and bottom (b) each coming in three 'colours'. The class of particles called leptons include the electron (e), but also five other particles – the electron neutrino (v_e), muon (μ), muon-neutrino (v_μ), the tau-particle (τ) and the tau-neutrino (v_τ). The mediator class contains the mediators of strong, weak and the electromagnetic interactions. These particles, together with the anti-particles of quarks and leptons, constitute the full list of elementary particles which have been observed so far. (We do not need to add anti-particles of the mediators in this list, as the photon, gluons and Z are there own anti-particles and $W^\pm$ are anti-particles of each other).

It turns out that this information can be incorporated into a sound mathematical theory, known as the **Standard Model.** The mathematical theory behind this model is known as **Yang Mills theory or non-abelian gauge theory.** So far standard model (SM)

has succeeded in explaining almost all observed experimental results. Recent observations of masses of neutrinos have necessitated a slight modification of the standard model, as according to the standard model, neutrinos must be massless. But this modification is a minor one, and is based on the same mathematical structure of non-abelian gauge theories. We shall now list the particle spectrum as predicted by the standard model :

Particle spectrum according to SM

Quarks:

$u_1, u_2, u_3 \quad d_1, d_2, d_3 \quad c_1, c_2, c_3$

$s_{1,}, s_2, s_3 \quad t_1, t_2, t_3 \quad b_1, b_2, b_3$

Leptons

(e, v_e), (μ, v_μ), (τ, v_τ)

Gauge bosons

Gluons: $g_1, \ldots, g_s$ Proton: γ

$W^\pm$, Z

Higgs ϕ

Note that this spectrum agrees almost completely with the experimentally observed spectrum. The only difference is the presence of a particle ϕ, called the Higgs, in the spectrum of the standard model. The existence of this particle is a definite prediction of the standard model which is yet to be varified experimentally, and currently a lot of experimental effort is devoted to finding this particle.

The non-abelian gauge theory underlying the standard model also tells us what kind of processes various particles can go through. Fig. 4 shows examples of some allowed processes, whereas Fig. 5 shows an example of a process which is not allowed in the standard model. Once we know the basic allowed processes, we can build up complicated processes with the help of these elementary processes. Example of a composite allowed process, obtained by combining the processes shown in Fig. 4 has been shown in Fig. 6.

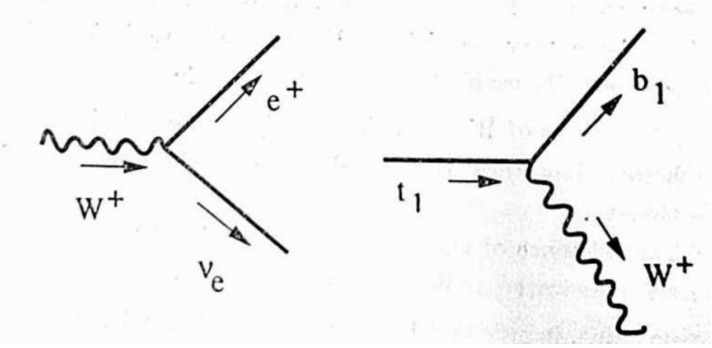

Fig. 4

Example of some allowed processes in the standard model

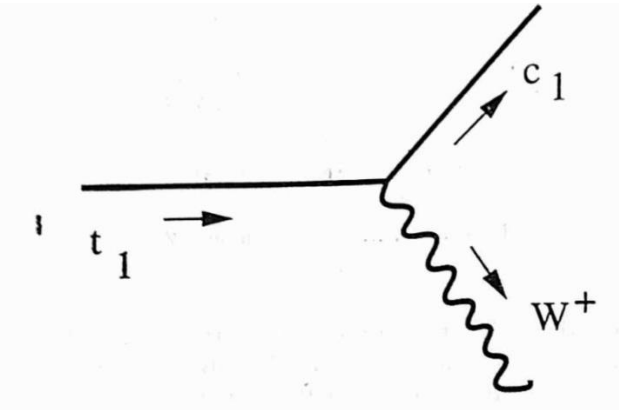

Fig. 5

Example of process which is not allowed
in the standard model.

One can explain almost all of the present day experimental results with the help of the standard model. With the help of this theory, we can also predict the result of any new experiment involving elementary particles. Many such predictions have been experimentally varified. Thus, for example :

- The masses of $W^{\pm}$ and Z particles were predicted long before they were discovered at the accelerator at CERN, Geneva.
- The existence of the t-quark was predicted long before it was discovered at Fermi lab near Chicago.

At this point, it may be illuminating to review why we know so much more now compared to the days of the Greek philosophers. As I see it, there are two main reasons :

1. With the advance of technology, we can now perform experiments which were not possible two thousand years ago. What does such an experiment involve? To understand this, let us note that our main aim is to find structure at small scale. In order to realize how

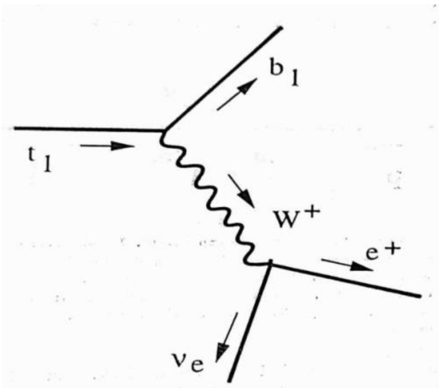

Fig. 6

Example of a composite allowed process in the standard model.

small these scales are, note that the size of an atom is of order 10^{-8} cm., the size of a nucleus is of order 10^{-13} cm., and the size of a proton or neutron is of order 10^{-14} cm. In order to see such small structures, we need light rays of small wave-length, at least as small as the distance we want to see. Using quantum mechanical wave-particle quality relations, we know that light of wave-length λ corresponds to a photon of energy hc/λ, where h is the Planck's constant and c is the velocity of light. Thus smaller wavelength corresponds to higher energy. In other words, in order to probe smaller and smaller structure, we need photons of higher and higher energy. Actually, instead of photon, we can use any other particle–a principle which is used in electron microscopes. Thus present day experiments in particle physics involve accelerating particles (proton, electron, etc.) to very high energies

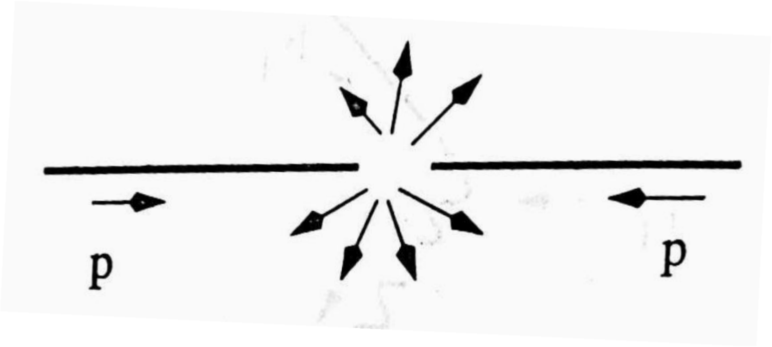

Fig. 7

A typical particle physics experiment. In this experiment protons are accelerated to high energy and are made to collide with each other. By measuring what comes out as a result of this collision, we can learn about internal structure of proton and properties of the quarks which make up the proton.

and making them collide with the particle whose structure we want to see. This has been schematically shown in Fig. 7. For this reason, elementary particle physics is also called high energy physics.

2. But experiments alone are not sufficient to account for the enormous success of today's particle physics. Much of the success is also due to the fact that there is a concrete mathematical theory underlying the standard model,–the non-abelian gauge theory. This is a simple generalisation of Maxwell's theory of electromagnetism. Recall that in classical electromagnetism, the basic fields are electric field $\vec{E}(\vec{x},t)$, and the magnetic field $\vec{B}(\vec{x},t)$. They satisfy Maxwell's equations. Non-abelian gauge theory is based on fields which are matrices instead of numbers. For example, the gluon fields (mediating strong interaction) are described by 3 × 3 matrices

$$\begin{pmatrix} \vec{E}_{11} & \vec{E}_{12} & \vec{E}_{13} \\ \vec{E}_{21} & \vec{E}_{22} & \vec{E}_{23} \\ \vec{E}_{31} & \vec{E}_{32} & \vec{E}_{33} \end{pmatrix} \text{ and } \begin{pmatrix} \vec{B}_{11} & \vec{B}_{12} & \vec{B}_{13} \\ \vec{B}_{21} & \vec{B}_{22} & \vec{B}_{23} \\ \vec{B}_{31} & \vec{B}_{32} & \vec{B}_{33} \end{pmatrix}$$

On the other hand, the $W^{\pm}$ and Z fields, mediating weak interaction, are described by 2 × 2 matrices.

$$\begin{pmatrix} \vec{\varepsilon}_{11} & \vec{\varepsilon}_{12} \\ \vec{\varepsilon}_{21} & \vec{\varepsilon}_{22} \end{pmatrix} \text{ and } \begin{pmatrix} \vec{\beta}_{11} & \vec{\beta}_{12} \\ \vec{\beta}_{21} & \vec{\beta}_{22} \end{pmatrix}$$

These matrix valued fields satisfy equations very similar to Maxwell's equations. This simple generalisation of Maxwell's equations, together with the principles of quantum mechanics, is responsible for all the success of standard mode.

> If we examine the gluon field, we see that there seem to be nine independent electric field components and nine independent magnetic field components, since a 3×3 matrix has nine entries. Should we not then expect nine gluous instead of eight? This apparent discrepancy is resolved by the fact that not all the components of the (3×3) matrix are taken to be independent. There is one single relation among the components, – the sum of the diagonal entries must vanish. Thus we have
>
> $$\vec{E}_{11} + \vec{E}_{22} + \vec{E}_{33} = 0, \ \ \vec{B}_{11} + \vec{B}_{22} + \vec{B}_{33} = 0.$$
>
> This gives only eight independent electric and magnetic field components and hence only eight gluons. A similar relation holds between the entries of the 2 × 2 matrices describing the $W^{\pm}$ and Z fields.

Does the success of standard model mean that we have finally found the ultimate theory that describes the world? The answer, unfortunately, is NO. First of all there is no guarantee that when we carry out the experiments at even higher energies, we'll not discover new particles, and or substructures of quarks, leptons, etc. In fact, many theorists have put forward proposals for new (mathematically consistent) theories where this happens. But there is a much more compelling reason why standard model cannot be the final story, This has to do with gravity.

Recall that we have ignored gravitational force in our discussion of elementary particles. In particular, standard model does not contain any particle that mediates gravitational interaction. But, however weak it may be, gravitational force is certainly present in nature. Thus our understanding of the world is not complete till we have a theory that described gravity as well. One can also consider thought experiments involving elementary particles where gravitational force becomes strong. For this, note that according to Einstein's theory of special relativity.

$$E = mc^2,$$

where E is the energy of a particle, m is the effective mass of the particle and c is the velocity of light in free space. As a result, we can increase the effective mass of an elementary particle by accelerating it to a very high energy. In this case the gravitational force between the elementary particles will become stronger and could be comparable to other forces.

Let us be more concrete. We have seen that the gravitational force between two protons is 10^{-36} times the electromagnetic force. But now consider accelerating both protons (in opposite direction) to such an extent that each of them carry energy equal to $10^{18}\, m_p c^2$, where m_p denotes the mass of a proton at rest. This increases the effective mass of each proton 10^{18} fold. As a result the gravitational force between them increases 10^{36} fold and becomes comparable to the electromagnetic force. Clearly, standard model cannot predict the result of this experiment. Unfortunately this experiment cannot be carried out in practice, due to the lack of accelerators powerful enough to accelerate the protons to such high energy. (At present we have succeeded in accelerating them to an energy of about $10^3 m_p c^2$). But a complete theory should be able to predict results of all experiments, even those that cannot be carried out due to practical difficulties.

Thus we see that despite its enormous success, standard model cannot be the final theory. The situation we are in today, is somewhat analogous to that of the ancient Greek philosophers. We are looking for the final theory that explains all natural phenomena, but we are unable to do the experiment that might provide a clue to this theory. However, there is one crucial difference! By our experience, we now know with reasonable degree of confidence that whatever the final theory is, there must be a mathematically consistent framework to describe it. So we can try to use this as a guideline in our search for the final theory.

It turns out that it is extremely difficult to construct a mathematically consistent theory that incorporates gravity, quantum mechanics and principle of relativity simultaneously. The problem comes from the fact that once we combine gravity and quantum mechanics, we are forced to introduce a new particle, – the graviton – which will mediate gravitational interaction. Elementary processes involving the graviton are dictated by the principles of relativity; and we can then consider composite processes obtained by joining together these elementary processes. It turns out that when we do that, some of the composite processes involving the graviton give rise to infinite answers. One such process has been shown in Fig. 8. In order to understand where this divergence comes from, let us view Fig. 8 as a series of events in space-time. Each space-time point where three lines meet, is known as the interaction vertex, since this is where the elementary interactions take place. Since these interactions can occur at any point in space-time, we need to sum over all such possibilities. This involves integrating over the locations of the space-time co-ordinates of these points. There are well-defined rules for calculating what the integrand should be, and one finds that the integrand becomes infinite when the three interaction vertices on the top half of the diagram come on top of each other. This is responsible for the divergence mentioned above.

This seems to be had news, but there is a good side to the story. What this teaches us is that if we manage to find a consistent theory of gravity, we should take it very seriously. In particular, if we find that there is only one such consistent theory, then it must be the final theory we are looking for.

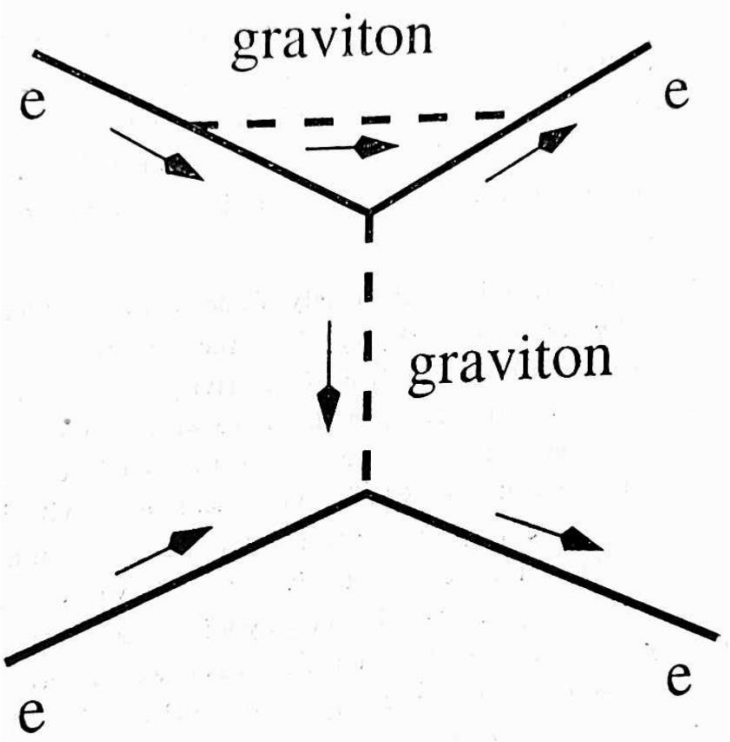

Fig. 8

A composite process involving the graviton which occurs in the process of scattering of two electrons. Here the dashed line denotes a graviton, and the solid line denotes an electron. There are well-defined rules for calculating the contribution of this process to *e*-*e* scattering amplitude, and one finds that this diagram gives an infinite answer.

It turns out that there is at least one consistent theory that can incorporate gravity, quantum mechanics and relativity. This theory is known as **string theory.** Whether it is the only theory which can do this is not known. But the fact that it incorporates gravity makes it a serious candidate for the final theory. So I shall devote the last few minutes of the talk to discussing the basic idea of string theory.

The basic idea in string theory is indeed quite simple. It says that different elementary 'particles', instead of being point-like object, are different vibrational modes of a string as shown in Fig. 9. This would seem to contradict the experimentally observed fact that elementary 'particles' like quarks and leptons appear point-like and not string-like. However, when one estimates the typical size of such a string, one finds that it is extremely tiny–of order 10^{-33} cm.

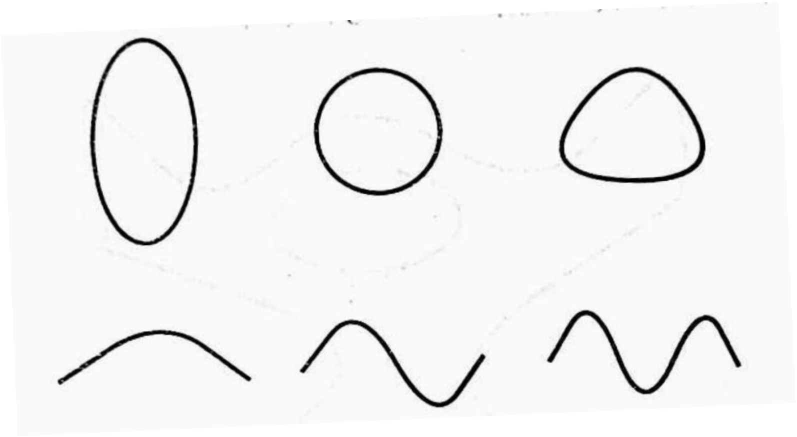

Fig. 9

Some vibrational modes of strings. Strings can come in two varieties – closed strings with no end, and open strings with two ends.

Present day accelerators do not have enough energy to see this structure (at present we can probe distances of order 10^{-17}cm.) and hence it is no surprise that these elementary objects corresponding to the vibrating strings appear to be point-like to present day experimentalists.

It turns out that one of these vibrational modes of the string corresponds to a particle that mediates gravitational interaction. Thus **string theory automatically contains gravity.** Further-more one finds that string theory does not suffer from any divergences. In order to understand the reason for this absence of divergence, let us examine how a diagram like the one shown in Fig. 8 looks like in string theory. Since here the electron as well as the graviton are expected to be vibrational modes of a string (let us say closed string) each line in the diagram will be fattened into the surface of a two dimensional cylinder, as shown in Fig. 10. This diagram looks like a set of cylinders joined together. Notice that in the limit when the radius of each cylinder goes to zero, we recover a diagram like the one shown in Fig. 8. However, upon closely examining the diagram of Fig. 10, we discover that this diagram has no interaction vertex; it is perfectly smooth everywhere! If there are no interaction vertices, then clearly there is no question of their coming close to each other and produce divergence.

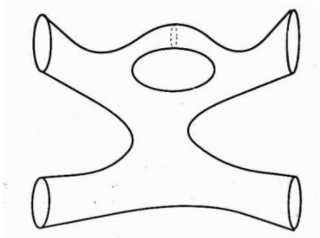

Fig. 10

String diagram showing the same scattering process, depicted in Fig. 8 , in string theory. Contribution from this diagram is free from any divergence.

This is the essential reason for finiteness of string theory. Of course, this general argument has been substantiated by many explicit calculations, showing the absence of divergence in string theory. Thus we see that **string theory gives a finite, relativistic, quantum theory of gravity!**

If we want to show that string theory is the final theory of nature, then we must also show that at low energy, when gravitational interactions are small, string theory can be approximated by standard model. Unfortunately, so far nobody has been able to show that is, although it has been shown that at low energy the theory approximately reduces to some Yang-Mills theory. Much of the present research in this subject is devoted to finding a proof that string theory, at low energy, can be approximated by the standard model.

It is now time to summarize the main points of the talk. They are as follows :

- At present we have an extremely successful theory, known as the standard model, that explains almost all experimental results involving elementary particles. This theory however does not include gravity and hence can not be the ultimate theory of all matter and their interactions.
- Finding a theory that explains all phenomena involving elementary particles, including their gravitational interaction, remains an open and challenging problem, although progress in this direction based on string theory has been encouraging.