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PREFACE

The Calcutta Institute of Theoretical Physics (CITP) is celebrating the 125th birth anniversary of its founder Prof. Kulesh Chandra Kar this year. This institute was set up by Prof. Kar in his residence in 1953 to hold meetings and discussions on a regular basis on new and emerging topics in physics. These meetings enabled young and aspiring physicists to have a forum beyond the daily grind of the regular learning and teaching process which are obviously essential for survival. While the disciplined learning process is absolutely necessary, Prof. Kar recognized, a century ago, that it is equally vital, equally demanding and in many ways immensely fulfilling to have an education beyond the daily grind. Hence the emergence of CITP. To celebrate the 125th birth anniversary of its founder, the institute organized a two-day event covering essentially all aspects of theoretical physics. The vital role of symmetry in the study of almost all physics related issues, the fundamental constituents of matter as envisaged by high energy physicists, the intriguing coming together of quantum physics and the classical world of gravity, the emergence of quantum computation, the contribution of S N Bose to a prediction that had to wait seventy years for experimental confirmation, the ever-fascinating world of astrophysics and the incredible mixing of biology with physics and mathematics formed the framework of a two day celebration. We do hope that the publication of the write-ups of the discussion will be enjoyed by those who were not actually present at the event.

Prof. Jayanta Kumar Bhattacharjee,

Director, CITP, Kolkata

C O N T E N T S

Symmetries and symmetry breaking

P. Mitra
Calcutta

Symmetry has been a key idea in particle physics.

First, a brief introduction.

- A symmetry is said to be present if an object has the same appearance after a change is carried out.
- We are familiar with reflection symmetry observed in nature and the rotational symmetry exhibited by many geometrical shapes.
- These symmetries, intuitively understood, can be made mathematically precise.
- A geometric figure is reflection symmetric about a straight line or a plane if one half, on reflection about the line or the plane, coincides with the other half. For example, the letter V.
- A geometric figure possesses a rotational symmetry about an axis if a rotation about that axis by a certain angle preserves the totality of the figure. Any regular polygon is an example. The rotation is about the axis cutting the plane orthogonally at the centre, through $2\pi/3$ in the case of an equilateral triangle Δ or $\pi/2$ for a square $\Box$. For a circle O, any angle of rotation yields a symmetry. Thus, symmetries may be continuous or discrete.
- In physics, reflections and rotations can certainly be symmetries, but there can be more general symmetries too. One has to specify what is being transformed, how and what remains unchanged under it.
- For example, if the velocity of a particle is rotated, its kinetic energy does not change. Hamiltonians may be rotationally invariant. The action may be invariant under Lorentz transformations. Maxwell's equations in free space are invariant when $\vec{E}$ is replaced by $\vec{B}$ and $\vec{B}$ by $-\vec{E}$.

- We shall mention some celebrated examples of symmetry and symmetry breaking in elementary particle physics.
- Like rotational invariance, parity invariance was also believed to hold in nature. Normal electromagnetic interactions certainly do not break parity invariance, which includes the invariance of Hamiltonians under the inversion of all space coordinates and corresponding changes of all fields.
- However, weak interactions break parity, as suggested by Lee and Yang and demonstrated soon thereafter. The Nobel prize was awarded to them in 1957.
- The idea of symmetry in quantum mechanics and in theories of particles and nuclei was developed to a great extent by Wigner, who won a Nobel prize in 1963. Apart from extensive studies of angular momenta, he is known for the theorem that a symmetry operation has to be represented by a unitary operator or an antiunitary one.
- Quantum electrodynamics, the quantum version of the gauge invariant electromagnetic theory, took a long time to develop because of technical problems. Feynman, Schwinger and Tomonaga won the Nobel prize for this in 1965.
- Developments in both theory and experiment followed in the sixties of the last century. Many short lived particles were discovered and classified by their quantum numbers manifested in the strong interactions.
- Gell-Mann was led to the quark structure of the particles and successfully predicted the existence of some new particle states. He won the Nobel prize in 1969. His discovery was the relevance of the SU(3) group in elementary particle physics.
- This is simply the group of unitary 3×3 matrices of unit determinant. Among the various quarks that occur, the u and the d are the lightest, and the ones present in the proton and the neutron. If heavier quarks and bound states made out of them are excluded, and the masses of the quarks ignored, an SU(2) symmetry is easy to spot.
- This is very closely related to the rotation group in a mathematical sense, though of course the transformation from one quark to another is physically very different from a rotation in three dimensional space. This symmetry is called the isospin symmetry in analogy with spin and has been very successful in nuclear physics.
- The extension of SU(2) to SU(3) becomes natural when the next quark, namely s, is included too. Its mass is higher, but to some approximation heavier quarks may be neglected and these three species of quarks considered together. The three quarks form the fundamental representation of SU(3), while many mesons, which are quark-antiquark bound states, were

seen to be in the adjoint octet representation and some triple quark states in a decuplet representation of the group.

- The gauge theory of electromagnetism involves U(1) gauge transformations, where a complex wave function or field may be multiplied by a phase factor, which is a U(1) or 1×1 unitary matrix.
- It was generalized to theories involving more complicated gauge transformations, like SU(2) or SU(3). This would require fields with internal SU(2) or SU(3) symmetry groups. Weinberg and Salam, following suggestions of Glashow, developed an SU(2)$\times$U(1) gauge theory to describe electroweak interactions. Doublet representations of SU(2) are provided by the quark pairs u and d, c and s, t and b, as well as by the lepton pairs. Glashow, Salam and Weinberg were awarded the Nobel prize in 1979 after elaborate tests of the model.
- The breaking of parity in the weak interactions, which was a surprise in the fifties, was soon built into theories of weak interactions involving a combination of vector and axial vector currents with equal strength. This makes parity violation maximal.
- But time reversal invariance was respected by these theories, and there was no experimental evidence to the contrary until the sixties. Cronin and Fitch were awarded the Nobel prize in 1980 for observing the breaking of this symmetry.
- The standard model including both electroweak theory and quantum chromodynamics with its SU(3) colour symmetry became established, but some puzzles remained. The major question was how the weak gauge bosons become massive and thereby lead to short range interactions. Apart from the weak gauge boson masses, the masses of other particles too were difficult to explain in the chiral theory of SU(2) involving only left handed doublets of quarks and leptons.
- These could be explained by a breaking of the SU(2)$\times$U(1) to U(1) in a special way: through the breaking of the symmetry of the ground state rather than directly of the interaction. This is called spontaneous symmetry breaking.
- Spontaneous breaking of symmetry had been studied by Nambu, for which he got a Nobel prize in 2008, shared with Kobayashi and Maskawa, who had studied the mixing of quarks and had noticed that the violation of time reversal needed the existence of at least three pairs or generations of quarks.
- A detailed theory of spontaneous breaking of gauge symmetry had been worked out in 1964 and was used in the subsequent construction of the standard model of electroweak interactions. It was after the sensational

2012 discovery of a particle which can be regarded as a relic of the spontaneous symmetry breaking of the electroweak gauge symmetry that the theoreticians Englert and Higgs were awarded the Nobel prize in 2013.

Next, we remind ourselves about **translation and rotation**.

Translation of an object in free space usually does not cause any change. A system may be described by a Lagrangian L which satisfies the equation of motion

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) = \frac{\partial L}{\partial q_i}.$$

Here q_i stands for a set of generalized coordinates. If a particular coordinate happens to be such that L is independent of it, the right hand side vanishes for that coordinate, implying that the corresponding time derivative on the left hand side vanishes, so that the partial derivative is constant. In other words, the generalized momentum conjugate to that q_i is constant.

When this cyclic coordinate q_i is a Cartesian coordinate, a change in its value is a translation and translation invariance of L implies conservation of the corresponding momentum. For example, one may have

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x, y).$$

In this case, V is independent of z and $m\dot{z}$ is constant.

A more interesting case occurs when there are two particles:

$$L = \frac{1}{2}(m_1\dot{\vec{x}}^2 + m_2\dot{\vec{y}}^2) - V(\vec{x} - \vec{y}).$$

There is no obvious cyclic coordinate, but L is invariant under

$$\vec{x} \to \vec{x} + \vec{a}, \quad \vec{y} \to \vec{y} + \vec{a},$$

where $\vec{a}$ is any constant vector. In such situations,

$$0 = \delta L = \sum_i \frac{\partial L}{\partial q_i}\delta q_i = \sum_i \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i})\delta q_i,$$

indicating that $\sum_i p_i \delta q_i$ is conserved, i.e., in this case $(m_1\dot{\vec{x}} + m_2\dot{\vec{y}})$ is conserved. This is the total momentum.

Another interesting case is when the cyclic coordinate is an angular variable. If one uses spherical polar coordinates, related to the Cartesian ones by

$$x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta,$$

a Lagrangian may happen to be independent of the angle ϕ. Consequently, the momentum conjugate to it, p_ϕ will be conserved. But

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) - V,$$

and

$$p_\phi = mr^2 \sin^2\theta \dot{\phi} = m(x\dot{y} - y\dot{x}),$$

which is the z component of the angular mommentum. All components of the angular momentum are conserved if the system is spherically symmetric.

Global symmetries refer to symmetries under rigid transformations, which may be internal, *i.e.,* resident in internal field space, or associated with space-time transformations like translations, rotations and more generally Lorentz transformations.

Suppose a field theory is characterized by a Lagrangian density $\mathcal{L}(\phi, \partial_\mu\phi)$ and it is invariant under some infinitesimal internal transformations of the fields ϕ

$$\phi \to \phi + \delta\phi = \phi + \epsilon\lambda\phi,$$

where ϵ is an infinitesimal parameter and λ is to be understood as a matrix acting on the column vector ϕ. The invariance means that

$$\delta\mathcal{L} = 0.$$

But

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\partial_\mu\phi.$$

Noting that δ and ∂_μ commute because of the internal nature of the transformation, and using the field equation, we find

$$\partial_\mu\big(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\phi\big) = \delta\mathcal{L}.$$

If the Lagrangian density is invariant, the right hand side vanishes and it follows that

$$\partial_\mu\big(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\lambda\phi\big) = 0.$$

This has the form of a conservation law

$$\partial_\mu J^\mu = 0,$$

and leads to a conserved charge

$$Q = \int d^3\vec{x} J^0 = \int d^3\vec{x}\pi\lambda\phi,$$

whose time-independence follows from the observation that

$$\frac{\partial}{\partial t}J^0 = -\frac{\partial}{\partial x^i}J^i$$

and the space integral of the divergence is equivalent to a surface integral at infinity, which can be set equal to zero with the understanding that all fields vanish at infinity.

This connection between a symmetry and a conservation law is generically referred to as Noether's theorem.

Note that the Poisson bracket

$$\{\phi, Q\} = \lambda\phi,$$

indicating that the charge is the generator of the symmetry transformation.

We have not specified what λ is. It has to be such that the transformation is a symmetry of the Lagrangian density. For example, if there are two scalar fields ϕ_1, ϕ_2 and the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{1}{2}m^2\phi_i\phi_i,$$

where the sum over i goes over the two values 1 and 2,

$$\delta\phi_1 = \epsilon\phi_2, \quad \delta\phi_2 = -\epsilon\phi_1$$

is a symmetry transformation. λ is a $2 \otimes 2$ antisymmetric matrix in this case.

If there are two symmetries, there will be two charges:

$$Q_1 = \int d^3\vec{x}\pi^T\lambda_1\phi, \quad Q_2 = \int d^3\vec{x}\pi^T\lambda_2\phi.$$

In this case, the Poisson bracket

$$\{Q_1, Q_2\} = \int d^3\vec{x}\pi^T[\lambda_1, \lambda_2]\phi,$$

as can be seen by using the fundamental Poisson bracket relations among the fields. Thus the commutator of the two λ-s appears in the Poisson bracket. The right hand side looks like a conserved charge, but it is not obvious at this point that it corresponds to a symmetry transformation. To see that it does, one has to note that the infinitesimal symmetry transformations corresponding to Q_1 and Q_2, namely

$$\delta\phi = \epsilon_1\lambda_1\phi, \quad \delta\phi = \epsilon_2\lambda_2\phi,$$

lead to finite transformations

$$\phi \to \exp(\theta_1\lambda_1)\phi, \quad \phi \to \exp(\theta_2\lambda_2)\phi,$$

where θ_1, θ_2 are finite parameters now. The commutator of two such transformations is the composite transformation

$$\phi \to \exp(\theta_1\lambda_1)\exp(\theta_2\lambda_2)\exp(-\theta_1\lambda_1)\exp(-\theta_2\lambda_2)\phi.$$

For small θ_1, θ_2, the change is

$$\delta\phi = \theta_1\theta_2[\lambda_1, \lambda_2]\phi.$$

This shows the commutator does indeed correspond to a symmetry transformation. Of course, the commutator may vanish, in which case the symmetry is trivial. If it does not, a new symmetry is produced from the two original symmetries and it is generated by the Poisson bracket of the two charges.

For fermions, the free Lagrangian density looks like

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi.$$

A special internal symmetry transformation is given by

$$\psi \to e^{-i\epsilon}\psi \approx (1 - i\epsilon)\psi,$$

with an infinitesimal parameter ϵ. This is of the general form considered in this section. The conserved charge is $\int d^3x\psi^\dagger\psi$, which is just the number operator, and follows from the conserved current

$$\bar{\psi}\gamma^\mu\psi.$$

If the fermion field comes in a $U(n)$ multiplet, ϵ can be a hermitian $n\times n$ matrix. Thus one may have a $U(n)$ symmetry. The current

$$\bar{\psi}\gamma^\mu T^a\psi$$

is conserved in the free theory. Here, T^a is a generator of the symmetry.

Another symmetry occurring in theories with fermions which have *no mass term* is given by

$$\psi \to e^{-i\epsilon\gamma^5}\psi \approx (1 - i\epsilon\gamma^5)\psi.$$

This is essentially a phase transformation, but the left and right handed components have different phases, hence it is called a chiral transformation. Mass terms, if present, break this symmetry, which is satisfied by kinetic terms as well as gauge interactions. The corresponding current conservation law is

$$\partial_\mu(\bar{\psi}\gamma^\mu\gamma^5\psi) = 0.$$

This equation holds at the classical level only and there is a correction in the quantum theory if gauge fields are present. This is called an *anomaly*.

Usually a mass term will be present. In that case, one finds that

$$\partial_\mu(\bar{\psi}\gamma^\mu\gamma^5\psi) = 2im\bar{\psi}\gamma^5\psi.$$

This is how the explicit breaking of the symmetry by the mass term manifests itself in a nonconservation equation.

The fermion field here has been assumed to have both left and right handed components. It is also possible to have a theory with just a left handed (or a right handed) fermion without mass. Then the current is of the form

$$\bar{\psi}_L \gamma^\mu \psi_L.$$

If the fermion field comes in a $U(n)$ multiplet, ϵ can be a hermitian $n \times n$ matrix. Thus one may have a chiral $U(n)$ symmetry

$$\psi \to (1 - i\epsilon^a T^a \gamma^5)\psi.$$

The current

$$\bar{\psi}\gamma^\mu \gamma^5 T^a \psi$$

is conserved in the free massless theory.

There are many other things one could talk about, but let us stop here now.

THANK YOU!

লেখাপড়া নিয়ে কিছু জরুরি কথা

অভিজিৎ কর গুপ্ত

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শোনা যাচ্ছে এবং দেখাও যাচ্ছে, গত কয়েক বছর ধরে কলেজ বিশ্ববিদ্যালয়ে পদার্থবিদ্যার-র মতো বেসিক সায়েন্স পড়তে ছেলেমেয়েরা অনিচ্ছা প্রকাশ করছে। ছাত্রছাত্রীদের সংখ্যা ক্রমাগত কমছে। দেখা যাবে, রসায়ন ও গণিতের ক্ষেত্রেও প্রায় একই অবস্থা। কোলকাতার কিছু কিছু বড় কলেজে এইসব বিষয়ে, বিশেষ করে পদার্থবিদ্যায়, অনার্স (বা মেজর) পড়ার জন্য এনরোলমেন্ট কমতে কমতে কোথাও কোথাও সংখ্যাটা এক অঙ্কে নেমে এসেছে, মফ:স্বল বা আসেপাশের ছোট কলেজে কোথাও কোথাও একেবারে শূন্য!

বিষয়টা নিয়ে এখনই মাথা ঘামানো দরকার। কী এমন হলো, যাতে বিষয়টা যেন একটা ক্রিটিক্যাল ফেজ ট্রানজিশান হওয়ার দিকে এগোচ্ছে?

ফিজিক্স-কেমিস্ট্রি-অঙ্ক-বায়োলজি এসব বেসিক সায়েন্স না পড়লে, এসব বিষয়ে নিরন্তর গবেষণা বজায় না রাখলে যে আমরা বিজ্ঞান ও প্রযুক্তির দিক থেকে দ্রুত পিছিয়ে পড়ব, তা আর বলার অপেক্ষা রাখে না।

এই সমস্যার সাথে সম্পৃক্ত আরো কয়েকটা বিষয় একটু খতিয়ে দেখা যাক। স্কুলের পাঠ শেষ করে বিজ্ঞানের ছাত্রছাত্রীরা যাচ্ছে কোথায়? পদার্থবিদ্যার মতো বেসিক সায়েন্স পড়তে না আসার প্রবণতা তৈরি হলো কেন? এইরকম ঘটনা কি এই রাজ্যের বাইরে দেশের অন্যত্রও ঘটছে? অন্য দেশেও কি এরকম ট্রেন্ড দেখা যাচ্ছে? সবকটারই উত্তর অল্পবিস্তর হলেও, হ্যাঁ।

তবে, আপাতত আমাদের রাজ্যের ছেলেমেয়েদের কথাই ভাবা যাক। স্কুলের পাঠ শেষে যাদের বিজ্ঞান পড়তে আসার কথা, তাদের মধ্যে একঝাঁক মেয়েরা চলে যাচ্ছে এখানে ওখানে গজিয়ে ওঠা কলেজগুলোতে নার্সিং পড়তে। নার্সিং পড়লে, একটা কিছু রোজগারের ব্যবস্থা হবে, এটা বাস্তব। এটাতো ঠিকই, আমাদের সংস্কৃতিতে ও সমাজব্যবস্থায় চাকরি না পেলে পড়াশুনার কোন মূল্য নেই। শুধু শুধু শিখে করবেটা কী? ভালোবেসে কোন বিষয় পড়তে আসা বা বুঝতে আসার দল সংখ্যায় নগন্য। একদল ছেলেমেয়েরা যাচ্ছে ইঞ্জিনিয়ারিং পড়তে। আজকাল, জয়েন্ট এন্ট্রান্স পরীক্ষা দিয়ে প্রায় এক লক্ষ র্যাঙ্ক করলেও কোন না কোন প্রাইভেট ইঞ্জিনিয়ারিং কলেজে পড়তে সুযোগ পাচ্ছে। হালে, এত বেশি সংখ্যক প্রাইভেট ইঞ্জিনিয়ারিং কলেজ এবং প্রাইভেট নার্সিং কলেজ তৈরি হয়েছে যে, সেসব জায়গায় ভর্তি হতে কোন অসুবিধা থাকছে না, পয়সা খরচা করতে পারলেই হলো।

যারা একটু পড়ুয়া, লড়াকু ছেলেমেয়ে (মেধাবী কথাটা বলব না ইচ্ছা করেই।) তারা আই আই টি-এর জন্য সর্বভারতীয় জয়েন্ট এন্ট্রান্স পরীক্ষা দিচ্ছে, তবে সেখানে তথাকথিত "দামী" বিভাগগুলোতে চান্স পাওয়া বেশ কঠিন, কারণ সিটের তুলনায় লড়াকু বা প্রত্যাশী ছেলেমেয়ের সংখ্যা অনেক বেশি। ওদিকে দেখা যাচ্ছে, ডাক্তারি পড়ার জন্য সর্বভারতীয় নীট পরীক্ষা

দেওয়ার প্রবণতা যেন অনেক বেড়েছে। এটা একটা নতুন স্যোসাল ওয়েভ। শহরে, গ্রামেগঞ্জে, মফস্বলে সর্বত্র ছেলেমেয়েরা ডাক্তার হওয়ার জন্য প্রচন্ড প্রত্যাশা নিয়ে তৈরি হচ্ছে। প্রতিযোগিতা তীব্র হচ্ছে। লড়াকু ছেলেমেয়েদের সাথে, সাধারণ ছেলেমেয়েরাও প্রবল প্রত্যাশা নিয়ে এগোচ্ছে। লাখো ছেলেমেয়েদের মধ্যে কিছু হাজার চান্স পাবে, বাকিরা একবার দুবার তিনবার চেষ্টা করবে, না পেলে বিফল মনোরথ হয়ে তারপর কোন একটা কলেজে এসে ভর্তি হবে। এই যে ডাক্তার হওয়ার জন্য প্রাণপন লড়াই, এখানেও অনেক হিসাব নিকাশ আছে, স্ট্রাট্রিজি তৈরি করার ব্যাপার আছে। এদের বেশিরভাগই বায়োলজি আর কেমিস্ট্রি ভালো করে পড়ে, কারণ নম্বর এখান থেকেই তুলতে হবে। ফিজিক্সে বেশি নম্বর উঠবে না, এটা ধরেই নেয়। আর, অঙ্ক নিলে তার পিছনে সময় ব্যয় করা স্ট্রাটেজির মধ্যে পড়ে না, শুনেছি কোচিং সেন্টারগুলোও এদের এই বুদ্ধি দেয়। কাজেই অঙ্কে ভীতিওলা ছেলেমেয়েরা, অথবা অঙ্কবর্জিত ছেলেমেয়েরা যখন পরে ডাক্তারিতে চান্স না পেয়ে কলেজে চলে আসে, তখন তাদের প্রথম প্রেফারেন্স কিছুতেই অঙ্ক নির্ভর বিষয়ের দিকে থাকবে না। বরং তারা বায়োলজি, জিওগ্রাফি এইসব বিষয় নিয়ে স্বাচ্ছন্দ্য বোধ করে, মন্দের ভালো হিসাবে।

এখন কথা হলো, ফিজিক্সে ভর্ত্তি হওয়ার উৎসাহ কমে যাওয়ার জন্য এসবই কি কারণ? নিশ্চয়ই আরো কিছু ব্যাপার আছে। ইদানীং আরেকটা ডামাডোল তৈরি হয়েছে, স্কুলের চাকরিতে নিয়োগ দুর্নীতি নিয়ে। একদল ছাত্রছাত্রীদের প্ল্যান ম্যাপ করা থাকে, গ্রাজুয়েশান, পোষ্টগ্রাজুয়েশান করে বিএড ট্রেনিং নিয়ে স্কুলের চাকরি করার জন্য তৈরি হওয়া। যদিও কুড়ি লক্ষ পরীক্ষা দিলে কুড়ি হাজারের ভাগ্যে শিঁকে ছিড়বে। মানে, এ এক লটারি পাওয়ার মত ব্যাপার! তবু, প্রত্যাশা ঝুলে থাকে সামনে, একটা বিশ্বাস কাজ করে, পরিশ্রম করে যায় তারা, যদি মিলে যায় একবার! এই আশায় যে এখন ভাটা পড়েছে তা তো বলাই বাহুল্য!

কিন্তু, এইসব সামাজিক ও মানসিক অবস্থার পরিবর্তন হলেই কি আবার ফিজিক্স পড়ার প্রবণতা বাড়বে? দেশে চাকরির জোয়ার বইলে, সরকারি নানান চাকরিতে নিয়মিত রিক্রুট হতে থাকলে, কলকারখানায় চাকরি বাড়লে কি আবার বিজ্ঞান পড়তে চাওয়া ছেলেমেয়েরা দলে দলে ফিজক্স পড়তে আসবে? আমার তা মনে হয় না।

প্রথম কথা, এই যে উপরে যা যা র ললাম, তা হওয়ার সম্ভাবনা খুবই কম। সরকারি চাকরি কমছে, আরো কমতে কমতে যে একদিন প্রায় শূন্য হয়ে যাবে না, তা বলা যাচ্ছে না। ওদিকে আধুনিক কলকারখানার গতিপ্রকৃতি, প্রোডাকশান মেথড পাল্টাচ্ছে, সেখানে লোক নিয়োগ কমতেই থাকবে। কারণ অটোমেশান, রোবট এবং কৃত্রিম বুদ্ধিমত্তার প্রভাব।

কর্পোরেটে যেসব চাকরি বেঁচে থাকবে তা মূলত: কিছু বিক্রিবাটা, মেইন্টেনান্স এবং সার্ভিস-এর কাজ। এসবের জন্য হায়ার ফিজিক্স বা খুব বেশি বেসিক সায়েন্সের জ্ঞানের দরকার হয় না।

আসলে, পড়ার আনন্দে পড়া, এই বিষয়টাকে যদি এখন আলোচনার মধ্যে না রাখি, তবে এটুকু দেখি যে, ছাত্রছাত্রীরা একটা বিষয় পড়তে গিয়ে শুরুতেই ভাবতে থাকে, এটা পড়ে আমার কী হবে, এতে কি কিছু স্কিল ডেভেলপমেন্ট হবে যা দিয়ে আমার চাকরি হবে? বিষয়টা উড়িয়ে

দেওয়ার মত নয়। একে তো গুরুত্ব দিতেই হবে। তবে, আমি ফিজিক্স পড়াতে গিয়ে ক্লাসে এই বিষয়ে কিছু আলোকপাত করার চেষ্টা করি। যেমন ধরা যাক, একটা ডিফারেন্সিয়াল ইক্যুয়েশান নিয়ে বলতে গিয়ে, তার সলভ করার মেথড নিয়ে আলোচনা করতে গিয়ে, তার এপ্লিকেশান দেখাতে গিয়ে, ইক্যুয়েশন-এর মধ্যে বিউটি ও তাকে এনালিসিস করার উপায় দেখাতে গিয়ে বলি, এই যে এতসব শিখলে, এতে আনন্দ পেলে, জ্ঞানলাভ হলো বটে কিন্তু, এটা শিখলেই কালকে এসব জানো বলে চাকরি পাবে না কিন্তু। তাহলে পড়বে কেন? এতে কি কোন স্কিল তৈরি হলো? অবশ্যই একটা মস্ত স্কিল তৈরি হলো! অঙ্ক ও বিজ্ঞানের এইরকম একটা মেথড শেখার সাথে সাথে তোমরা শিখলে কী করে একটা কঠিন সমস্যার সমাধান করতে হয়, কিভাবে ভাবতে হয়, কিভাবে স্টেপ বাই স্টেপ এগোতে হয়। এর থেকে বড় 'স্কিল' আর কী হতে পারে? বস্তুত, এভাবে যে জ্ঞানের স্তর তৈরি হয়, চিন্তা-ভাবনার অভ্যাস তৈরি হয়, প্রবলেম সলভ করার স্কিল তৈরি হয়, নতুনভাবে কিছু করার একটা স্পৃহা তৈরি হয়, তা নিশ্চয়ই এই ফিজিক্স পড়ুয়াদের অনেকদূর এগিয়ে রাখবে।

শুধু বিজ্ঞান নয়, যেকোন বেসিক সাবজেক্টই একটু গভীরে গিয়ে দেখলে, মন দিয়ে ভালোবেসে পড়লে তা আমাদেরকে অনেকদূর এগিয়ে দিতে পারে। এই বিশ্বাস রাখাটাও জরুরি। একটা জিনিস বোঝা যাচ্ছে, চাকরির বাজার আরো উদ্বায়ী (ভোলাটাইল) হতে থাকবে, আর সেই নিত্য পরিবর্তনশীল বাজারে নিজেকে টিঁকিয়ে রাখার উপায় হলো নিজেকে সময়মতো নতুন করে আবিষ্কার করা। এই প্রসেসে আমাদের বেসিক নলেজ আর গভীর ভাবনার ক্ষমতাই হলো আসল স্কিল!

মুস্তাফা সুলেইমান-এর কথা ধরা যাক। সিরিয়া থেকে তাদের ফ্যামিলি ইংল্যান্ডে এসেছিলেন আগেই। মুস্তাফা ইউ কে-এর নাগরিক, ওখানকার একটা বিশ্ববিদ্যালয়ে দর্শন নিয়ে পড়াশুনা করলেন তিনি। পড়াশুনা শেষে নিজেকে প্রশ্ন করতে থাকলেন, এরপর কি? কি নিয়ে আমাদের ভাবা উচিৎ এই মহুর্তে? কোনদিকে এগোনো উচিৎ? ভেবে দেখলেন, AI বা কৃত্রিম বুদ্ধিমত্তা বিষয়ে জানবেন, তা দিয়ে অনেক আপাতত অধরা কিছু সমস্যার উত্তর বার করবেন। ভাবনাটা খুব আন্তরিক হলে, সমাধান সম্ভব। এই বিশ্বাস থেকে কয়েকজন বন্ধু মিলে তৈরি করলেন Deep Mind কোম্পানি। আমরা সকলেই জানি এই ডীপমাইন্ড কোম্পানি পরে কত অসাধারণ প্রবলেমের সমাধান করেছিলো, কত বিলিয়ন ডলার মুনাফা করেছিলো। পরে অবশ্য গুগুল এই ডীপমাইন্ড-কে কিনে নেয়। আমরা অবাক বিস্ময়ে দেখতে লাগলাম, গুগুলের এই ডীপমাইন্ড কৃত্রিম বুদ্ধিমত্তার নানান অভিনব এলগোরিদম কাজে লাগিয়ে কিভাবে একের পর এক অসাধ্য সাধন করছিলো। গো-এর মত অত্যন্ত জটিল বোর্ড গেম বুঝে ফেলে অপ্রতিরোধ্য হিউম্যান চ্যাম্পিয়ানদের হারিয়ে দেওয়া, আর তারপর আলফাফোল্ড দিয়ে দীর্ঘদিনের অপরাজেয় দুর্ভেদ্য এক জটিল বিষয়, প্রোটিন ফোল্ডিং-এর সমাধান করে ফেলা, এসবই সম্ভব হয়েছে মুস্তাফা সুলেইমানের বন্ধু ডীপমাইন্ডের ডেমিস হাসাবিসের নেতৃত্বে। ডেমিস হাসাবিস এইবার কেমিস্ট্রিতে তাঁর আলফাফোল্ড-এর কাজের জন্য নোবেল পুরষ্কার পেলেন। ডেমিস কিন্তু, কম্পিউটার সায়েন্সের লোক। আরো একটা উদাহরণ হলো, জিওফ্রে হিনটন, তিনিও কম্পিউটার সায়েন্সের মানুষ অথচ নোবেল পুরষ্কার পেলেন ফিজক্সে। হিনটনও গুগুলে

ছিলেন এবং কিছুকাল আগে গুগুল থেকে বেরিয়ে এসেছিলেন। এখানে দুটো জিনিস লক্ষ্যনীয়। অন্য সাবজেক্ট থেকে এসে ফিজিক্স এবং কেমিস্ট্রিতে নোবেল পুরষ্কার পাওয়া এবং গুগুলের মতো কর্পোরেট থেকে নোবেল পাওয়া। এই ব্যাপারগুলো নিয়ে আমাদের ফিজিক্স কমিউনিটিকে, ইউনিভার্সিটি, কলেজের অধ্যাপক ও কর্তাব্যক্তিদের ভাবতে হবে বৈকি। আগামীদিনে আমরা কিভাবে ফিজিক্স পড়বো বা পড়াবো তাই নিয়ে সিরিয়াস আলোচনা হওয়া দরকার।

কথা হলো, শত শত বছর ধরে চলে আসা এই যে কলেজ বিশ্ববিদ্যালয়ের শিক্ষাব্যবস্থা, এই নলেজ সিস্টেম তা কি একইরকম থাকবে নাকি দ্রুত কোন পরিবর্তনের দিকে যাবে? আমরা কি প্রস্তুত তার জন্য?

দেখা যাচ্ছে, নতুন এডুকেশান পলিসিতে হরেক বিষয়ের বন্যা বইয়ে দেওয়া হয়েছে। সেমিস্টার সিস্টেম, অজস্র বিষয় ও তার নামকরণ। মেজর, মাইনর, এস ই সি, এম ডি সি/ আই ডি সি, ভ্যালু এইডেড সাবজেক্ট, এ ই সি সি, ডি এস ই, এসব নামের আড়ালে অনেকরকম বিষয়ের সাথে পরিচিত করিয়ে দেওয়ার একটা উদ্দেশ্য হয়ত আছে, কিন্তু, আদতে যে ছেলেমেয়েরা কোনকিছুই ঠিক করে শিখে উঠতে পারছে না, তা তো খালি চোখেই দেখতে পাচ্ছি। কিছু নাম অবশ্যই শিখছে, কিছু মুখস্থ করছে, এদিক থেকে ওদিকে হরেকরকম ক্লাসের জন্য ছোটাছুটি করছে, কিন্তু, দিনের শেষে সবকিছু থেকে কোন স্কিল তৈরি হচ্ছে কি? কিছু ভাবনাচিন্তা করার অভ্যাস তৈরি হচ্ছে কি? বিষয়ের উপর দক্ষতা, কনফিডেন্স বাড়াতে পারছি কি আমরা? এই প্রশ্নগুলো করা এখন বিশেষ জরুরি।

জেনেছি, ইঞ্জিনয়ারিং কলেজগুলোতেও ইঞ্জিনিয়ারিং সাবজেক্টগুলোর বেস তৈরির জন্য যে ফিজিক্স ও অঙ্ক শেখানো হয় প্রথমদিকে, তার গুরুত্ব কমানো হচ্ছে। কারণ হয়ত তাদের মনে হয়েছে, ওসব শিখিয়ে লাভ হবে না। তারচেয়ে কিছু ফর্মূলা আর কিছু নাম শিখিয়ে দিলে ওরা দিনের শেষে ইন্টারভিউতে উৎরে যাবে, তারপর কোম্পানি বুঝে নেবে। এভাবেই সর্বনাশ ডেকে আনছি আমরা।

আমরা যদি ভাবতে বসি যেভাবে ফিজিক্সের কারিকুলাম ডিজাইন করে এসেছি সেভাবেই চলবে? আমার মনে হয়, এখানে আমাদের বিশেষ গুরুত্ব দিয়ে ভাবতে হবে। আগামীদিনের নিউ ওয়েভ কি? মনে করা যাক তা কৃত্রিম বুদ্ধিমত্তা ও ডাটা সায়েন্স এবং সিন্থেটিক বায়োলজি। এসব একজায়গায় গিয়ে মিলছে। অসম্ভব সব কাজ হচ্ছে আর আমাদের সোসাইটিতে, আমাদের সংস্কৃতিতে, পড়াশোনায় তার প্রভাব পড়তে বাধ্য। এখন প্রশ্ন হলো, আমাদের চিরপুরাতন বেসিক সায়েন্স-এর বিষয়গুলোকে কি এইসব দিকে একটু ওরিয়েন্ট করা যায়? আমরা কি কোয়ান্টাম মেকানিক্স এমনভাবে পড়াতে পারি যাতে ছেলেমেয়েরা কোয়ান্টাম ইনফরমেশন, কোয়ান্টাম কম্পিউটিং এইসব দিকের গবেষণায় বা ইনোভেশানে সহজেই অংশগ্রহণ করতে পারে?

AI এবং ডাটা সায়েন্স নিয়ে চর্চা করতে গিয়ে দেখেছি, এইসব এরিয়াতে যা দরকার হয়, তা হলো লিনিয়ার এলজেবরা, একটু ক্যালকুলাস, একটু স্ট্যাটিস্টিক্স-এর জ্ঞান আর এলগোরিদম বোঝার ক্ষমতা, পাইথন-এর মতো সহজ একটা কম্পিউটার ল্যাঙ্গোয়েজ শিখে ফেলা। ব্যাস! এর থেকে বেশি কিছু নয়। এসব বিষয়ের অনেকটাই তো বেসিক সায়েন্স, বিশেষ করে ফিজিক্সের কর্মকান্ডের সাথে সংপৃক্ত। তাহলে, আমরা কি পারবো নতুনরকম কিছু ভাবতে?

মূল কথা হলো, বেসিক সায়েন্সের চর্চায় গলদ থাকলে বিজ্ঞানের ছাত্রছাত্রীদের পক্ষে ক্যারিয়ার তৈরি করা, ইনোভেশান করা বা নিজের অস্তিত্বকে টিঁকিয়ে রাখা কঠিন হবে। অন্যদিকে দেশও রসাতলে যাবে। শুধু উপর উপর কিছু জেনে আর টেকনোলজি কিনে একটা জনজাতির বা একটা দেশের পক্ষে আন্তর্জাতিক বাজারে মাথা উঁচু করে টিঁকে থাকা সম্ভব নয়। অন্যদিকে নতুন নতুম টেকনোলজির সাথে সাথে আমাদেরকে এগোতে হবে। কাজেই সেখানেও আমাদের অংশগ্রহণ করতে গেলে বেসিক সায়েন্স, বিশেষ করে ফিজক্স-এর মতো সাবজেক্ট কে ওরিয়েন্ট করতে হবে।

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Quantum Gravity

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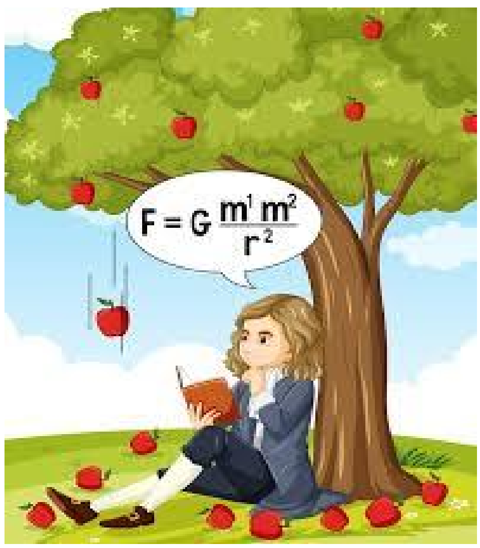
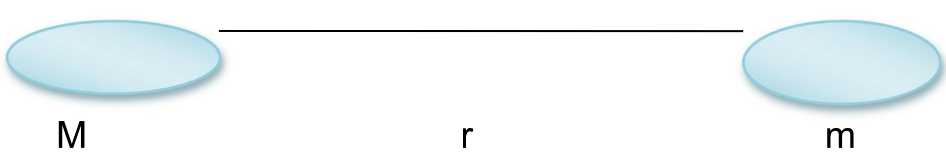
Newton proposed the force of gravitation as a force acting between two masses kept at a distance. It was a remarkable description of action at a distance and it is believed that such an idea came to his mind when he observed an apple falling downward from the tree. In absence of any obvious reason for this downward motion of the apple, Newton conjectured that earth being a massive body attracts the apple towards it with a force which he named ‘the force of Gravitation’.



To describe the force quantitatively, he obtained the quantitative description of the magnitude of the force as



$$\mathbf{F = G\,Mm / R^2}$$

Where M and m are the two point-like masses (Though the figure above doesn't look like that), R is the distance between them and G is a constant called Universal gravitational constant.

The universality of the law as well as the value of 'G' was remarkably demonstrated when this law was applied successfully to describe the planetary motion around the sun which was earlier proposed by Keplar using extensive and valuable experimental data found by Tycho Brahe.

Problem with Newtonian Gravity

According to Newton the gravitational force or interaction propagates instantaneously. If mass M moves, gravitational force on m at a distant point changes instantaneously. Hence m can sense the motion of M at the same instant when M has moved. Thus the speed of propagation of gravitational effect is infinity i.e. it is instantaneous. This contradicts a strongly established finding that no causal effect an propagate faster than speed of light. Einstein's Special Theory of Relativity which is established on strong observational and theoretical support is based on the proposition that no information can propagate faster than the speed of light. So far there is no observation which contradicts the laws of Special Relativity. Thus, despite it's huge success, Newtonian gravity cannot be an accurate description of force of gravitation. Instead of Newtonian concept of gravitational force, Einstein came out with a brilliant idea. According to Newton, the space is flat (endowed with Euclidean Geometry) and a body travels along the shortest path i.e. straight line in absence of any external force. However, in presence of force the body accelerates and moves in a curved path in space- time. These are the essence of Newton's Laws of Motion.

straight line in space - time

$$x = ut \text{ (NO ACCELERATION)}$$

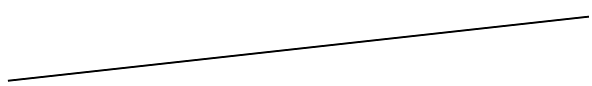
curved line in space- time

$$x = ut + at^2/2$$

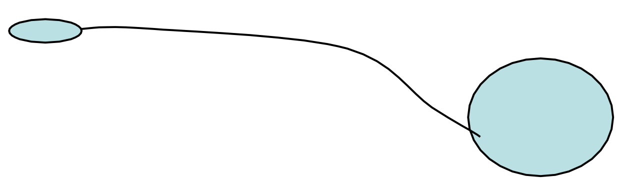
$$\text{(ACCELERATION = } a \text{)}$$

Newtonian picture

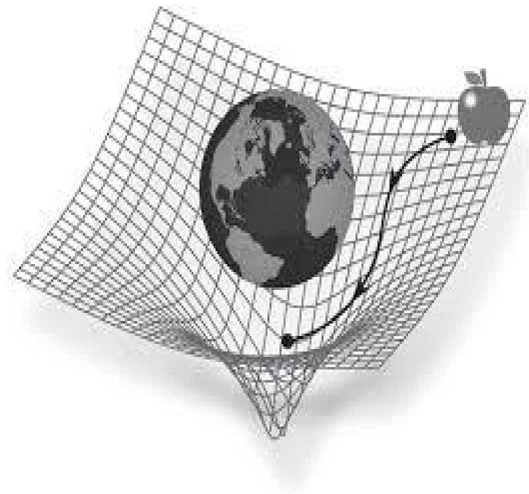
In absence of force the path in space-time is a straight line



In presence of force , the path in space-time is a curved line



Einstein's picture – space time curvature



Force and Geometry

Presence of mass (energy) curves the space-time (as shown in the figure above).The shortest distance is now a curved line giving the illusion of acceleration and hence the existence of gravitational force. Thus the illusion of force originates from curved geometry.

Flat geometry

In flat Euclidean geometry there always exists a coordinate in which the shortest distance between two points

$$(x,y,z) \quad \text{and} \quad (x + dx,\ y + dy,\ z + dz) \quad \text{is}$$

$$ds^2 = dx^2 + dy^2 + dz^2$$

Curved geometry

In curved spacetime the shortest distance is,

$$ds^2 = (1+ f_1(x,y,z))\, dx^2 + (1 + f_2(x,y,z))\, dy^2 + (1 + f_3(x,y,z))\, dz^2$$

where the coefficients are never (1,1,1) like flat geometry. These extra quantities ‘f ‘ measures the curvature of the space and determine whether the space is flat or curved.

Four Dimensional Space-Time

According to Einstein, the time is also like a coordinate for the description of an event and therefore distance between two points in four dimensional curved geometry is (Taking c = 1)

$$ds^2 = (1 + f_1)\, dx^2 + (1 + f_2)\, dy^2 +$$
$$(1+ f_3)dz^2 - (1 + f_4)\, dt^2$$

These coefficients of the differentials are called metric of the space-time.

Riemannian geometry

From this metric we construct various tensor quantities

$R_{\alpha\beta}\ (f_1, f_2, f_3, f_4)$, R or even higher rank tensors

which measure the curvature of the space-time. They depend on the metric and their derivatives.

If $f_1, f_2, f_3, f_4 = 0$ then all the curvature functions are zero which implies flat space-space-time.

Einstein's Equation:

We write this famous equation in a simple and symbolic way,

$$\mathbf{R = k\,M}$$

R measures curvature, M measures mass (energy) and k is a constant proportional to the universal constant 'G' .

Thus, Mass (energy) curves space-time. A particle travels in a curved line in such a space-time and we imagine that there is a force called Gravity. This theory is called the General Theory of Relativity.

In flat space in Cartesian coordinates :

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2$$

However, the flat space in polar coordinates:

$$ds^2 = dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\Phi^2 - dt^2$$

solve Einstein's equation in polar coordinate for solar mass 'M' to determine:

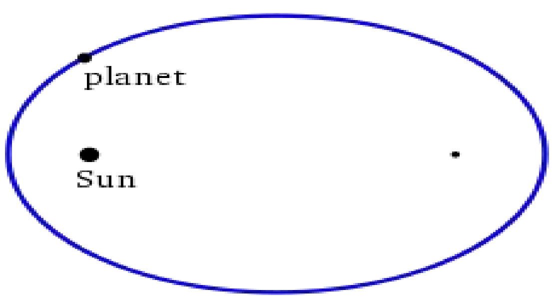
f_1, f_2, f_3, f_4

Schwarzschild geometry for mass M

$$D^2 = (1 - 2GM/r)^{-1}\, dr^2 - (1 - 2GM/r)\, dt^2$$

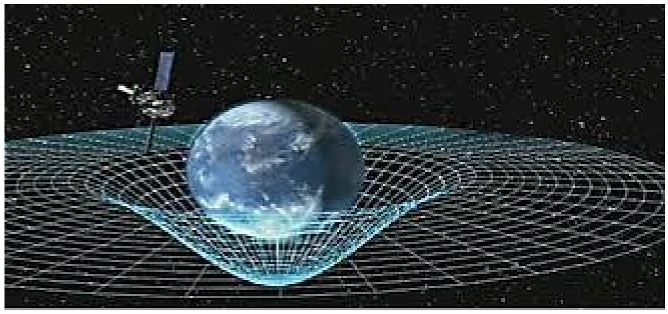
$$+ r^2\, d\theta^2 + r^2 \sin^2\theta\, d\Phi^2$$

M = 0, Space Time Is Flat new definition of mass/energy makes space-time curved and inducts effects similar to gravitational force tests of Einstein's picture consider solar mass and solve Einstein's equation to find the path for a planet. The Result is an Ellipse!!



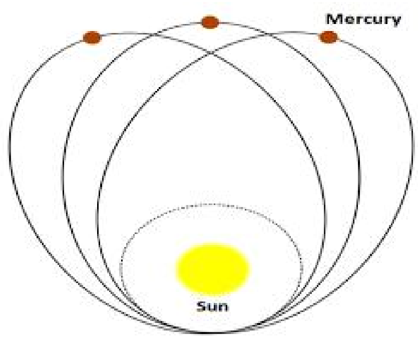
But this is same as Keplar's law of planetary motion as explained by Newton's law of gravitation.

Curved Space Time



Something more !!

Einsteins theory predicts something more than just elliptical motion.it predicts perihelion precession of the planetary orbits as shown in figure below:



$$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

Complete agreement

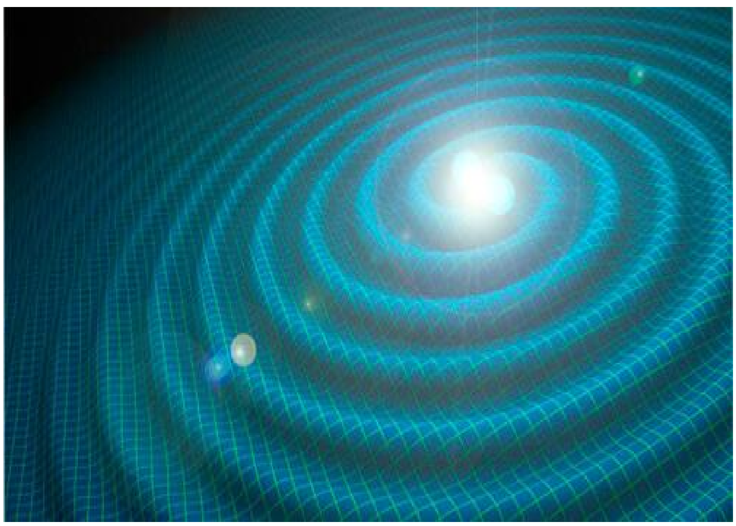
From Einstein's theory, the angle of rotation for the planet Mercury = .011 degree per century. This exactly matches with the experimental data.

Subsequently the bending of path of light was tested with experiment to confirm the validity of Einstein's description of force of gravity through space-time geometry.

Gravitational wave equation

Using Einstein's equation, the metric coefficients 'f' as described earlier are found to satisfy the wave equation below, This according to Einstein's theory the movement of mass causes space-time curvature to fluctuate like the surface of an ocean which travels like a wave at speed c = 300000000 m/sec.

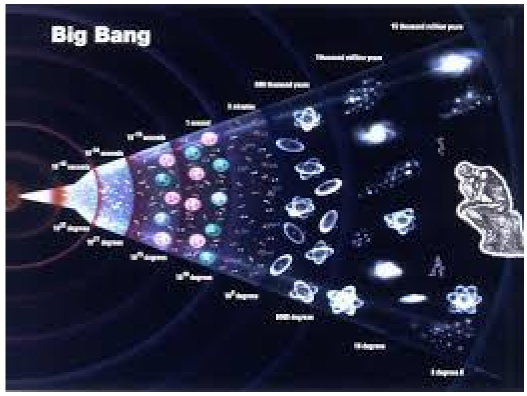
Gravitational wave subsequently detected experimentally in LIGO detector to confirm the above finding and ensures that it doesn't travel faster than light as was found in Newton's law of Gravitation.



Gravitational Wave

Cosmology

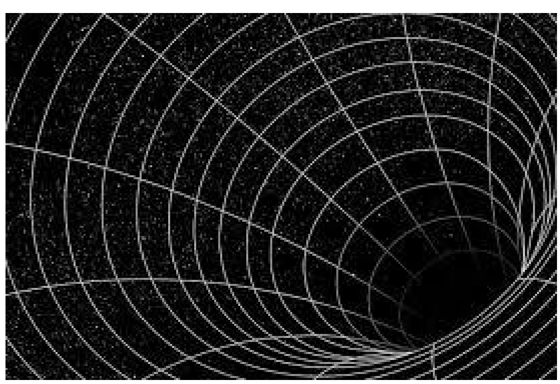
Einstein's General relativity is successfully then applied to understand the birth and expansion of the universe following Hubble's Law. Hubble found that our visible universe is expanding with time and if we go back in time then at some initial time t = 0 the whole universe focusses into a point. Thus from this point. due to a huge explosion called big bang, about 14 billion years ago, the universe started to grow.



BLACK HOLES

From Einstein's equation, due to space-time curvature, a star may collapse into a very dense state such that the spacetime around it become viciously curved and even light can escape when it reaches up to a distance from the centre of the mass distribution. The surface defining this distance is called horizon and this state of space-time is called Black hole.

HIGH CURVATURE LEADS TO BLACK HOLE



Recall that for a mass M,

$$ds^2 = (1 - 2GM/r)^{-1} dr^2 - (1 - 2GM/r) dt^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$$

For $r < 2GM$, the signatures of the coefficients of r and t are reversed and like time r can change only along one direction!! Thus, for a Black hole, the radial coordinate of an in-falling particle always decreases.

Once Inside $r = 2GM$, anything (Even Light) Can travel only in one direction!! All paths (geodesics) converge to origin $r = 0$ and ends lead to Black hole singularity.

$R = 2GM$ is called the horizon radius of black hole. Evidences of existence of black holes put Einstein's theory on strong footing that the Gravitational force is manifestation of Space-time geometry.

Questions

Is Einstein's theory final theory of gravity?

Is it consistent with Quantum theory?

Classical theory

In classical mechanics fundament al law is Newton's law:

$$\text{Force} = m \; d^2x/dt^2$$

It's solution gives position x of the mass m at time t as,

$$x = x(t)$$

Quantum Theory

According to Quantum Theory:

For microscopic particles we can not find the path exactly. Instead we can find a function $\psi(x, t)$ which gives probability or chance to find the particle at the point x at time t.

This probability function satisfies Schrodinger equation $H\,\psi = i\, h/2\pi\, d\psi/dt$ (H is the Hamiltonian or total energy operator). The solution is $\Psi(t) = :\exp[-i/(h/2\pi) \int h\, dt]\, \psi(t = 0)$

Semiclassical theory

Trea t force ----Classically

Particle -- Quantum mechanically

STEPHEN HAWKING

Semiclassical Theory

Black hole are treated as classical gravity and the in-falling particles are treated following quantum mechanics.

Is black hole really black?

(a semiclassical effect!)

For flat space in polar coordinate:

$$ds^2 = dr^2 - dt^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$$

For a star of mass M the geometry

$$ds^2 = (1 - 2GM/r)^{-1} dr^2 - (1 - 2GM/r) dt^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$$

Singularity

$$ds^2 = (1 - 2GM/r)^{-1} dr^2 - (1 - 2GM/r) dt^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\Phi^2$$

(Take G = 1 in some appropriate unit)

At r = 0 the metric blows up and the theory fails. This is expected since the source M is sitting at r=0. However, the singularity at r=2M is unexpected.

A metric component becomes infinity implies that the physics breaks down? But why it should happen at r = 2M?

It is because of wrong choice of coordinates!!

Change coordinate -- no singularity at r = 2M.

Wha t happens near horizon?

Consider a point very close to horizon (near horizon).

$r = 2M + x^2/8M$ (x is very very small)

As $x \rightarrow 0$, $r \rightarrow 2M$ i.e. the horizon.

Neglecting higher powers of x, we have,

$$D^2 = -(kx)^2\ dt^2 + dx^2 + (d\theta^2 + \sin^2\theta\ d\Phi^2)/4k^2$$

$(k = 1/4M)$

Near horizon 2-dimensional space-time

$$D^2 = -(kx)^2\ dt^2 + dx^2$$

Now remember 2-dim flat space has no singularity:

$$ds^2 = dx^2 + dy^2 \quad \text{(cartesian)}$$
$$ds^2 = r^2\ d\theta^2 + dr^2 \quad \text{(polar)}$$

Euclideanize the time $t \rightarrow i t = \theta$

$$D^2 = x^2\ d(k\theta)^2 + dx^2$$

No singularity -----flat space with radial

Coordinate x and angular coordinate $k\theta$ with period 2π

Period of euclideanized time θ is $2\pi/k$.

Consider quantum theory near the horizon

$$H\psi = i\ (h/2\pi)\ d\psi/dt$$

$$\Psi(t) =: \exp(-i/(h/2\pi) \int H\, dt)\ \Psi(t = 0)$$

Evolution operator: $\exp(-i/(h/2\pi) \int H\, dt)$.

Integra ting over $i t = \theta$ over full period

$$\exp(-2\pi/hk\ H)$$

This is Similar to Boltzman factor exp (-E/ k BT) for a thermal bath. (kB is Boltzmann Constant) Comparing, we find that the Temperature

$T=hk/2\pi kB \rightarrow$Hawking temp

The horizon of the black hole in a singularity free coordinate

Exhibits a thermal character with a characteristic temperature

$$T = hk/2\pi k_B \quad \textbf{(A QUANTUM ORIGIN)}$$

As k=1/ 4 M, Hawking temperature decreases with mass of the black hole.

For twice solar mass Hawking temperature is~nano-kelvin.

(less than background cosmic microwave ~ 3 k) -- so invisible.

As the black hole swallows mass / energy it's temperature decreases implying a negative specific heat and more it swallows it's horizon area keeps on increasing just as entropy.

The entropy of the black hole is identified as,

$$s = A/4$$

A = area of the horizon

Using these, we find that these quantities have similar relations among themselves just as laws of thermodynamics

$$k\,dA = dM + w\,dJ$$
$$dA > 0 \text{ etc.}$$

Before going inside the blackhole the objects have definite information about their characteristics (ordered). Due to it's temperature what may come out from black hole (as radiation) is a thermal radiation

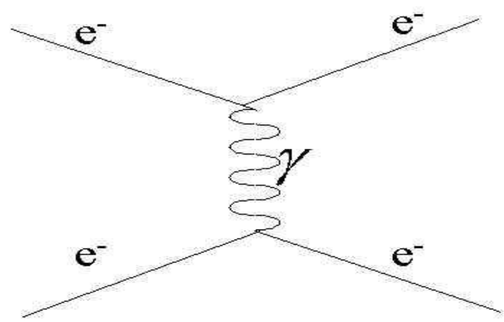
(disordered) implying information loss!

Quantum description of Force

Instead of semiclassical theory we must consider quantum theory of the full system. For this we need to know: quantum theory of gravity. Our quantum understanding of force between two particles are exchange of some force mediating particles (called the quanta of force)

For example

Force between electrons



Force Quanta:

1. electromagnetic force ---- photon
2. weak nuclear force ---- w+, w-, z
3. strong nuclear force --- 8 gluons
4. gravitational force – gra viton

However to describe quantum mechanically, the last one namely gravity brings in A famous problem called Quantum gravity problem.

According to quantum theory exchange of electromagnetic force is exchange of a particle (quanta) called photon. What is the similar description for gravitation in the language of space-time geometry proposed by Einstein?

Quantum aspects of gravity- a longstanding problem

For gravitation the spacetime curvature is described by a fluctuation of geometry about the flat space-time:

$$D^2 = (1 + f_1)\, dx^2 + (1 + f_2)\, dy^2 + (1 + f_3)\, dz^2 - (1 + f_4)\, dt^2$$

$$g_{\mu\alpha} = \eta_{\mu\alpha} + \sqrt{G}\ h_{\mu\alpha} \quad , \quad (\ h_{\mu\alpha} \rightarrow \text{Graviton}\ )$$

Note that **G** has dimension **(mass)** $^{-2}$
A scalar – gravity coupling is given by,

$$g^{\mu\alpha}\ \partial_\mu\ \Phi\ \partial_\alpha\ \Phi = \partial_\mu\ \Phi\ \partial^\mu\ \Phi + \sqrt{G}\, h^{\mu\alpha}\ \partial_\mu\ \Phi\ \partial_\alpha \Phi$$

Second term in the right hand side gives the graviton-scalar coupling.

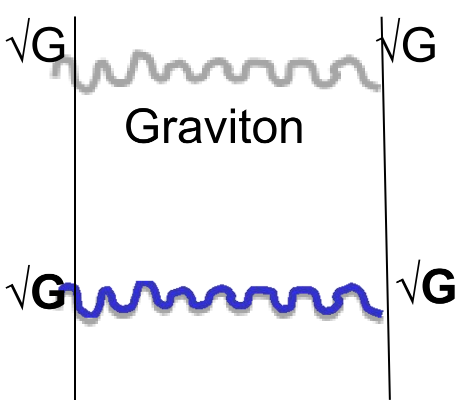
Amplitude for the graviton exchange:

Graviton

~ $(E^2\ G)$ (IN UNIT h = 1, c =1) Where E is centre of mass energy. This implies that quantum effects of Gravity is relevant at energy scale.

$E \sim 1/G^{1/2} = 10^{19}$ GeV called Planck energy
(M_P) we are only at 10^3 Gev!! (LHC)

Can ignore graviton exchange effects but why is it a problem?



LOOP DIAGRAMME

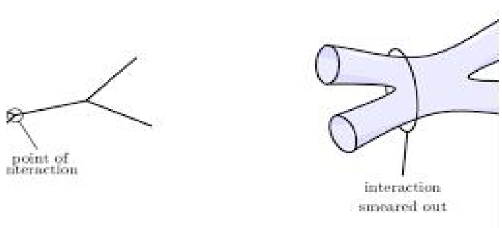
2-GRAVITON EXCHANGEAMPLITUDE ~ $\Lambda^4 / M_P{}^4$

The amplitude is UV divergent and non-renormalizable.

Einstein's gravity is incomplete at high energy (short distance) and needs correction due to Quantum effects.

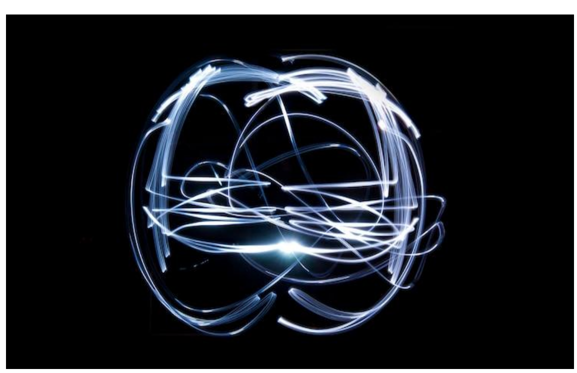
String Theory

Fundamental entities are not point-like particles but one dimensional strings which interact as the figure below. This helps to remove divergences.



Loop Quantum Gravity

Space-time is not continuum but described by discrete spin network. This idea also offers a possible resolution to avoid divergences.



Each of these concepts bring in some additional questions whose complete answers are not known yet

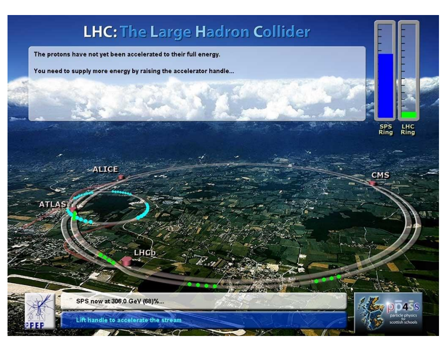
Laboratory at sky

Hubble Space Telescope

Laboratory on Earth

Large Hadron Collider



Our search to understand the deeper mysteries of Space-time continues to unveil the story of this beautiful Universe.

Physics Education Research – An Overview

Partha Pratim Roy
Center for Physics Education Research, Kolkata
South Point High School, Kolkata

Physics teaching and learning at different levels, from high schools to colleges and universities, are facing enormous challenges in recent times. There is a continuous decrease in the number of enrollments in physics courses and, even if students are enrolled, they are not showing interest in attending classroom lectures. When asked, students complain about the archaic nature of the physics curricula, traditional lecture based teaching methods, non-availability of jobs for physics graduates, domination of engineering and computer science based skills in methods of physics research, etc. How can the physics community address the problem and find a solution? Here comes the importance of 'Physics Education Research' or simply PER, an interdisciplinary field for studying such problems and searching for solutions. How students learn physics and assimilate the concepts for future applications is the question that triggered the emergence of this field. Improvement and updating of the existing curriculum, introduction of scientific pedagogical techniques for better learning outcomes, development of innovative assessment and evaluation tools, linking physics based skills to jobs, etc. are necessary to address the problems mentioned above. Findings from Physics Education Research show that general educational research is not sufficient for solving physics specific instructional and curricular issues. Therefore, PER is necessary for addressing issues in physics education, like Mathematics Education Research is for issues in mathematics education.

Physics Education Research (PER) is a recognized field of research like High Energy Physics, Condensed Matter Physics, Materials Physics, Astrophysics, Biophysics etc. in the Physics departments of universities globally. In the USA, there are very active Physics Education Research groups at University of Maryland College Park, University of Colorado, The Ohio State University, University of Illinois, North Carolina State University, Rutgers University, University of Washington Seattle, Arizona State University, Kansas State University, and many others. Also in Europe there are PER groups at Edinburgh University, University of Glasgow, Uppsala University, Goethe University, Ludwig Maximilian University, Technical University of Munich, and many others, in their physics departments. In India, dedicated research groups of

PER exist only at the Homi Bhaba Centre for Science Education of TIFR. Center for Physics Education Research Kolkata, India, is an initiative of some physics educators of Kolkata to study curricular and instructional issues of physics education in West Bengal. There is enough scope of research on physics education in India, where science education in general and physics education in particular are in a very sorry state, and remedial actions are badly needed. However, most of the physicists here think that by virtue of being researchers of other areas of physics they are automatically qualified to talk about physics education research without any training in this field. This approach should change, and we need to take a professional approach to PER. There should be a formal process of training in this field if we are serious about addressing the problems.

There exists dedicated journals for publishing research articles on PER. '*Physical Review Physics Education Research*' is the journal of the renowned *Physical Review* series. PER section of the '*American Journal of Physics*', '*Physics Education*' of the European Physical Society, '*The Physics Teacher*' of the American Association of Physics Teachers (AAPT) are the most impactful among the PER community. In addition, PER papers are also published in journals of Education, Psychology, Cognitive Science, viz. '*Cognition and Instruction*', *Journal of The Learning Sciences*, *Journal of Research in Science Teaching*, *Journal of Science Education*, and the like. In March 2024, the famous journal *Nature Physics* published a focus issue on Physics Education Research to emphasize the importance of this field of research. In India, however, physics related societies are not very serious about publishing any such journal. *Physics Education,* published by the Indian Association of Physics Teachers, and *Physics Teacher*, published by the Indian Physical Society, publishes articles on different fields of physics, but not on physics education. If there are some articles on physics education, they are mostly opinions, not research. The Indian Academy of Sciences publishes *Resonance, the journal of science education*, which contains articles of science written for high school and undergraduate students by experts in that field. A Journal of Science Education is expected to publish research articles on science education, not just undergraduate version of the topics of advanced research. To help improve the state of physics education in particular and science education in general, in our country, our scientists need to bridge the gap between two different perspectives - the perspective of pedagogy, psychology and sociology, and the perspective of disciplinary authenticity and content.

Methods of Physics Education Research are mostly the methods used in educational, psychological and anthropological research. Neurological techniques like fMRI (functional Magnetic Resonance Imaging) are also used for collecting data. The methods can be quantitative, qualitative or a mixture of both. Quantitative methods use statistical analysis of survey data and in qualitative method analysis of transcripts of interviews with students, video of activities like problem solving sessions, discussion sessions etc. are done. For quantitative research, concept-survey tests are developed for data collection. Force Concept Inventory (FCI) is one such test that has had enormous impact in undergraduate teaching of basic Newtonian mechanics. FCI is being used since 1995 to measure concepts of force and motion among high school students and undergraduates. FCI was developed by David Hestenes, Malcolm Wells, and Greg Swackhamer in 1992 at the Department of Physics of The Arizona State University, and was later modified by Richard Hake and Ibrahim Halloun in 1995. Eric Mazur of the department of physics, Harvard University, had used FCI to measure the Newtonian concepts of force and motion among his students. He was shocked to discover that 70% of his students were Aristotelian even after completing his course on Newtonian mechanics with excellent grades. To remedy the situation, he developed a method of teaching what he called "Peer Instruction". Learning of the concepts had improved considerably after using this new technique. He had published his findings in his papers on PER.

Qualitative research is inductive in one hand and subjective in other. The data collected by a qualitative researcher are non-numerical and are in the form of statements of opinions, understandings, beliefs, pictures, videos etc. Analysis of these data are made by finding words or group of words communicating a specific state of mind or a specific kind of concept, and several hypotheses are made based on this analysis. This part of the research is subjective in the sense that the way the researcher understands the issue influences the hypothesis to be made. The research questionnaire prepared for collecting data is also influenced by subjective viewpoint of the researcher. Testing of the hypotheses using quantitative method is finally necessary to draw an objective inference about the problem. Any research of significance should have to be both qualitative and quantitative.

Developing concept inventories is an important trend in physics education research. Equally important is the analysis of the data obtained after using the inventories. Other than FCI, there are tests like MMCE (Mathematical Modelling Conceptual Evaluation), VET (Vector Evaluation Test), FMCE (Force Motion Concept Evaluation), HTCE (Heat Temperature Concept Evaluation), ECCE (Electrical Circuits Concept Evaluation) – all developed by Ron Thornton of Tufts University and David Sokoloff of University of Oregon, together. There are also tests like ECS (Energy Concept Survey) by Chandralekha Singh of University of Pittsburgh, DIRECT (Determining and Interpreting Resistive Electric Circuits Concept Test) by Paula V. Engelhardt of Tennessee Tech University and Robert J. Beichner of North Carolina State University, which are developed and used for purposes of research on conceptual foundations of students on specific topics. Based on the results of these surveys, pedagogical and curricular interventions are proposed. Devising models of intervention is also another important trend in PER. "Peer Instruction" of Eric Mazur is the outcome of such interventional research after using FCI.

How to create an active learning environment is also a trend in PER. SCALE-UP (Student Centered Active Learning Environment with Upside-down Pedagogies) is such a project developed by Dr. Robert Beichner of North Carolina State University in mid-1990s, and it prescribes a kind of classroom design appropriate for active learning. As many as 500 institutions including MIT has adopted SCALE-UP in undergraduate classrooms. Eugenia Etkina and her PER group of Rutgers University invented ISLE (Interactive Science Learning Environment) for creating active learning environment using experiments in high school and undergraduate classrooms. Observational Experiments, Testing Experiments and Application Experiments are the three pillars of ISLE. After making observations and measurements, students represent their observations through diagrams, graphs, tables etc. and form a hypothesis to explain what they observed. To test whether their hypothesis is tenable or not, students design a Testing Experiment. If the hypothesis is not rejected, students devise application experiments to see how the hypothesis can be used to innovate devices for everyday applications. The process of active, discovery-based learning is closely related to philosophical issues related to pedagogy. Research on personal epistemology, primitive concepts and beliefs are important trends in this direction. Andrea diSessa of University of California, Berkeley was a leader in this trend of PER. A deep understanding of the concepts of physics is necessary to begin research in this direction. Frederick Reif of University of California, Berkeley is also a trendsetter in this direction. His famous text on Statistical Physics is an evidence of that.

Physics Education Research is done in collaboration between the departments of Physics and Education. PhD students can get degrees either in Physics or in Education, depending on the department in which the candidates have taken major courses. What are the major courses necessary to begin with PER? As we need training in quantum field theory (QFT), statistical mechanics, critical phenomena, techniques of simulation, etc. to enter condensed matter physics, a physics education researcher is required to take courses on research methodology, cognitive psychology, epistemology, etc. in addition to the courses like quantum mechanics, QFT, Statistical Mechanics, etc. Without a grasp over foundations of any topic of physics how can a researcher investigate what is happening in a learner's mind when the learner is trying to understand that topic? While others use QFT, Stat-Mech etc. to investigate natural phenomena, the physics education researcher creates mind-map of those subjects as a student learns them. The growth of the textbooks by Griffiths, e.g. Introduction to Quantum Mechanics, is the result of such research. Without compromising with the authenticity and rigor of the subject, the learners' process of thinking is always kept at the centre of such research. In this era of artificial intelligence and machine learning, PER can develop algorithms to devise a teaching guide for teachers and a learning guide for students to incorporate individual learner's characteristics in the learning and teaching process. Data from quantitative and qualitative research can help build such algorithms for different topics of physics. Classroom teaching as well as remote teaching of physics have been transformed significantly by PER in several famous universities worldwide, and a lot more this research can contribute to both the process and product of physics departments, if proper emphasis and funding continue in this field.

References:

1. F. K. Richtmyer, "Physics is Physics," American Physics Teacher (which became American Journal of Physics) 1 (1), 1-5 (1933).
2. J. M. Rice, The Public-School System of the United States (Century, New York, 1983).
3. Richard P. Feynman, Surely You're Joking, Mr. Feynman (W. W. Norton & Company, New York, 1985).

4. National Commission on Excellence in Education, A Nation at Risk: The Imperative for Educational Reform (National Academy Press, Washington DC, 1983).
5. Committee on Prospering in the Global Economy of the 21st Century,Rising Above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future (The National Academies, Washington DC, 2006).
6. Robert Beichner, Jeff Saul, David Abbott, Jeanne Morse, Duane Deardorff, Rhett Allain, Scott Bonham, Melissa Dancy, and John Risley,"Student-Centered Activities for Large Enrollment Undergraduate Programs (SCALE-UP) project," in PER-Based Reform in University Physics, edited by E. F. Redish and P. J. Cooney (American Association of Physics Teachers, College Park, MD, 2007), Vol. 1.
7. Paula Heron and David Meltzer, "The Future of Physics Education Research: Intellectual Challenges and Practical Concerns," Am. J. Phys.73 (5), 390-394 (2005).
8. Available at http://www.umit.maine.edu/~wittmann/FamilyTrees.jpg, accessed July 7, 2008.
9. Seymour Papert, Mindstorms: Children, Computers, and Powerful Ideas (Basic Books, New York, 1993), 2nd. ed.
10. For a list, visit http://www.physics.umd.edu/perg/homepages.htm or see http://www.PER-CENTRAL.org.
11. Robert Beichner, Richard Hake, Lillian McDermott, Jose Mestre, Edward Redish, Frederick Reif, and John Risley, "Support of Physics Education Research as a Subfield of Physics: Proposal to the NSF Physics Division," (1995). Available from http://www.ncsu.edu/per/Articles/NSFWhitePaper.pdf, accessed July 9, 2008.
12. David Hestenes, Malcolm Wells, and Gregg Swackhamer, "Force Concept Inventory," Phys. Teach. 30 (3), 141-158 (1992).
13. Ibrahim Halloun and David Hestenes, "The Initial Knowledge State of College Physics Students," Am. J. Phys. 53 (11), 1043-1055 (1985).
14. Ibrahim Halloun and David Hestenes, "Common Sense Concepts About Motion," Am. J. Phys. 53 (11), 1056-1065 (1985).
15. Eric Mazur, Peer Instruction: A User's Manual (Prentice-Hall, Upper Saddle River, NJ, 1997).
16. Catherine H. Crouch and Eric Mazur, "Peer Instruction: Ten Years of Experience and Results," Am. J. Phys. 69 (9), 970-977 (2001).
17. Richard Hake, "Interactive-engagement Versus Traditional Methods: A Six-thousand-student Survey of Mechanics Test Data for Introductory Physics Courses," Am. J. Phys. 66 (1), 64-74 (1998).
18. American Physical Society, "APS STATEMENT ON RESEARCH IN PHYSICS EDUCATION," in APS News (College Park, Maryland, 1999), Vol. 8.
19. David Meltzer, Lillian McDermott, Paula Heron, Edward Redish, and Robert Beichner, "A Call to the AAPT Executive Board and PublicationsCommittee to Expand Publication of Physics Education Research Articles within the American Journal of Physics," (2003).
20. David E. Trowbridge and Lillian C. McDermott, "Investigation of Student Understanding of the Concept of Velocity in One Dimension.," Am. J. Phys. 48 (12), 1020-1028 (1980).

21. David E. Trowbridge and Lillian C. McDermott, "Investigation of Student Understanding of the Concept of Acceleration in One Dimension.," Am. J. Phys. 49 (3), 242-253 (1981).
22. Available on line at <http://www.ipn.unikiel.de/aktuell/stcse/stcse.html>, accessed Aug 12, 2008.
23. David Hammer, "Student Resources for Learning Introductory Physics," Am. J. Phys. 68 (S1), S52-S59 (2000).
24. Jonathan Tuminaro and Edward F. Redish, "Elements of a Cognitive Model of Physics Problem Solving: Epistemic Games," Phys. Rev. ST PER 3 (2007).
25. Young-Jin Lee, David J. Palazzo, Rasil Warnakulasooriya, and David E. Pritchard, "Measuring Student Learning with Item Response Theory," Phys. Rev. ST PER 4 (1), 010102 (2008); Lei Bao and Edward F. Redish, "Model Analysis: Representing and Assessing the Dynamics of StudentLearning," Phys. Rev. ST PER 2 (010103), 16 (2006).
26. R. Padraic Springuel, Michael C. Wittmann, and John R. Thompson, "Applying Clustering to Statistical Analysis of Student Reasoning About Two-dimensional Kinematics," Phys. Rev. ST PER 3 (2), 020107 (2007).
27. Lillian C. McDermott and Edward F. Redish, "Resource Letter: PER-1: Physics Education Research," Am. J. Phys. 67 (9), 755-767 (1999).
28. Located at http://prst-per.aps.org/
29. See http://www.aapt.org/Events/newfaculty.cfm for more information.
30. http://www.aps.org/units/fed/newsletters/spring2008/redish-brooks.cfm
31. http://www.PER-CENTRAL.org
32. A. B. Arons, Teaching Introductory Physics (John Wiley and Sons, New York, 1997).
33. E. F. Redish, Teaching Physics with the Physics Suite (John Wiley & Sons, Hoboken NJ, 2003).
34. Randy Knight, Five Easy Lessons: Strategies for Successful Physics Teaching (Addison Wesley, San Francisco, 2002).
35. K. A. Ericsson and H. A. Simon, Protocol Analysis: Verbal Reports as Data (MIT Press, Cambridge, MA, 1993).
36. Michelene T. H. Chi, "Quantifying qualitative analyses of verbal data: A practical guide," J. Learn. Sci. 6 (3), 271-315 (1997).
37. V. Otero, Hammer, D., and May, D., workshop on "How Do We Know What Students Are Thinking: Theory and Methods in Video Data Analysis," in Summer Meeting of the American Association of Physics Teachers (Rochester, NY, 2001);
38. Valerie Otero, "Qualitative Research on Student Conceptual Development: The Need for a Theoretical Framework", in Enrico Fermi Summer School in Physics Education Research (Varenna, Italy, 2003).
39. Rebecca Lippmann Kung, Peter Kung, and Cedric Linder, "Equipment Issues Regarding the Collection of Video Data for Research," Phys. Rev. ST PER 1 (1), 010105 (2005).
40. A list of some of the more popular assessments is available at http://www.ncsu.edu/per/TestInfo.html. A CD accompanying Redish's "Physics Teaching with the Physics Suite" contains 17 concept surveys.

41. Jim Minstrell, “Explaining the "at rest" Condition of an Object,” Phys.Teach. 20 (1), 10-14 (1982).
42. R. J. Beichner, “Testing Student Interpretation of Kinematics Graphs,” Am. J. Phys. 62 (8), 750-762 (1994).
43. Beth Ann Thacker, “Recent Advances in Classroom Physics,” Reports on Progress in Physics 66 (10), 1833-1864 (2003).
44. P. R. L. Heron, M. E. Loverude, P. S. Shaffer, and L. C. McDermott, “Helping Students Develop an Understanding of Archimedes' Principle. II. Development of Research-based Instructional Materials,” Am. J. Phys. 71 (11), 1188-1195 (2003);
45. Stamatis Vokos, Peter S. Shaffer, Bradley S. Ambrose, and Lillian C. McDermott, “Student Understanding of the Wave Nature of Matter: Diffraction and Interference of Particles,” Am. J. Phys. 68 (S1), S42-S51 (2000);
46. Karen Wosilait, Paula R. L. Heron, Peter S. Shaffer, and Lillian C. McDermott, “Addressing Student Difficulties in Applying a Wave Model to the Interference and Diffraction of Light,” Am. J. Phys. 67 (S1), S5-S15 (1999);
47. Bradley S. Ambrose, Peter S. Shaffer, Richard N. Steinberg, and Lillian C. McDermott, “An Investigation of Student Understanding of Single-slit Diffraction and Double-slit Interference,” Am. J. Phys. 67 (2), 146-155 (1999);
48. Lillian C. McDermott and Peter S. Shaffer, “Research as a Guide for Curriculum Development: An Example from Introductory Electricity. Part I: Investigation of Student Understanding,” Am. J. Phys. 60 (11), 994-1003 (1992);
49. Lillian C. McDermott and Emily H. van Zee, presented at the International Workshop: Aspects of Understanding Electricity, Ludwigsburg, West Germany, 1984 (unpublished).
50. Ronald K. Thornton and David R. Sokoloff, “Assessing Student Learning of Newton's Laws: The Force and Motion Conceptual Evaluation and the Evaluation of Active Learning Laboratory and Lecture Curricula,” Am. J. Phys. 66 (4), 338-352 (1998).
51. Douglas Huffman and Patricia Heller, “What Does the Force Concept Inventory Actually Measure?” Phys Teach 33 (3), 138-143 (1995).
52. Andrea DiSessa, “Knowledge in Pieces,” in Constructivism in the Computer Age, edited by G. Forman and P. Pufall (Lawrence Erlbaum Associates, Hillsdale, NJ, 1988), pp. 49-70.
53. D. E. Brown and David Hammer, “Conceptual Change in Physics,” in International Handbook of Research on Conceptual Change, edited by S. Vosniadou (Routledge, New York, 2008), pp. 127-154.
54. Dewey Dykstra, C. Franklin Boyle, and Ira A. Monarch, “Studying Conceptual Change in Learning Physics,” Sci. Ed. 76 (6), 615-652 (1992).
55. John Clement, “Students' Preconceptions in Introductory Mechanics,” Am. J. Phys. 50 (1), 66-71 (1982).
56. George J. Posner, Kenneth A. Strike, Peter W. Hewson, and William A. Gertzog, “Accommodation of a Scientific Conception: Toward a Theory of Conceptual Change,” Sci. Ed. 66 (2), 211-227 (1982).
57. David Hammer, “Epistemological Considerations in Teaching Introductory Physics,” Sci. Ed. 79 (4), 393-413 (1995);

58. Edward F. Redish, "Implications of Cognitive Studies for Teaching Physics," Am. J. Phys. 62 (9), 796-803 (1994);
59. Edward F. Redish, "Theoretical Framework for Physics Education Research: Modeling Student Thinking," in Proceedings of the International School of Physics, "Enrico Fermi" Course CLVI, edited by Edward F. Redish and M. Vicentini (IOS Press, Amsterdam, 2004);
60. Rosemary Russ, Rachel Scherr, David Hammer, and Jamie Mikeska, "Recognizing Mechanistic Reasoning in Student Scientific Inquiry: A Framework for Discourse Analysis Developed from Philosophy of Science," Sci. Ed. 92, 499-525 (2008).
61. Andrea A. diSessa and Bruce L. Sherin, "What Changes in Conceptual Change," Int. J. Sci. Ed. 20 (10), 1155-1191 (1998).
62. Michelene Chi and James Slotta, "Ontological Coherence of Intuitive Physics," Cog. Inst. 10 (2/3), 249-260 (1993).
63. Philip Johnson-Laird, How We Reason (Oxford University Press, New York, 2006).
64. Dedre Gentner and Albert Stevens (eds), Mental Models (Lawrence Erlbaum, Hillsdale, NJ, 1983).
65. David P. Maloney, "Research on Problem Solving: Physics", in Handbook of Research in Science Teaching and Learning, edited by D. Gabel (MacMillan Publishing Company, New York, NY, 1993) pp. 327- 354.
66. Leonardo Hsu, Eric Brewe, Thomas Foster, and Kathleen A. Harper, "Resource Letter RPS-1: Research in Problem Solving," Am. J. Phys. 72 (9), 1147-1156 (2004).
67. Michelene T. H. Chi, Paul J. Feltovich, and Robert Glaser, "Categorization and Representation of Physics Problems by Experts and Novices," Cog. Sci. 5 (2), 121-152 (1981).
68. Patricia Heller and Mark Hollabaugh, "Teaching Problem Solving Through Cooperative Grouping. Part 2: Designing Problems and Structuring Groups," Am. J. Phys. 60 (7), 637-644 (1992); Patricia Heller, Ronald Keith, and Scott Anderson, "Teaching Problem Solving Through Cooperative Grouping. Part 1: Group Versus Individual Problem Solving," Am. J. Phys. 60 (7), 627-636 (1992).
69. Chandralekha Singh, "Assessing Student Expertise in Introductory Physics with Isomorphic Problems. I. Performance on Nonintuitive Problem Pair from Introductory Physics," Phys. Rev. ST PER 4 (1), 010104 (2008); Laura N. Walsh, Robert G. Howard, and Brian Bowe, "Phenomenographic Study of Students' Problem Solving Approaches in Physics," Phys. Rev. ST PER 3 (2), 020108 (2007).
70. W. K. Adams, K. K. Perkins, N. S. Podolefsky, M. Dubson, N. D. Finkelstein, and C. E. Wieman, "New Instrument for Measuring Student Beliefs about Physics and Learning Physics: The Colorado Learning Attitudes about Science Survey," Phys. Rev. ST PER 2 (1), 010101 (2006).
71. Giulia Polverini and Bor Gregorcic, "Performance of ChatGPT on the test of understanding graphs in kinematics", Phys. Rev. Phys. Educ. Res. 20, 010109 (2024).
72. Luiza Vilarta Rodriguez, Jan T. van der Veen, Ton de Jong, "Role of analogies with classical physics in introductory quantum physics teaching", Phys. Rev. Phys. Educ. Res. 21, 010108 (2025).

73. Cassandra A. Paul and David J. Webb, “Examining equity and graduation rates at two institutions using a course deficit model and the collaborative learning through active sense-making in physics curriculum”, Phys. Rev. Phys. Educ. Res. 21, 010107 (2025).
74. Gerd Kortemeyer and Wolfgang Bauer, “Cheat sites and artificial intelligence usage in online introductory physics courses: What is the extent and what effect does it have on assessments?”, Phys. Rev. Phys. Educ. Res. 20, 010145 (2024).

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